Feasibility Conditions of SIR-based Power Control in TDMA Wireless Systems

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Abstract — In this paper, the problem of SIR-based power control in narrowband wireless systems is revisited. Results on feasibility conditions of SIR-based power control are given. We stated and proved the relations between the achievable SIR and the link gain matrix in a narrowband system. Taking into account the orders of the realistic values for the problem quantities and assuming the desired SIR is chosen properly, then there exist infinite many solutions of the SIR-based power control problem. Otherwise, there is hardly a solution since the existence conditions are very restrictive. It is noticeable that these feasibility conditions have important implications on systems capacity and can be applied to call admission control (CAC). Based on the feasibility conditions, an improved stepwise removal algorithm (ISRA) is proposed for TDMA systems to achieve a better performance.

I. REVIEW OF SIR BASED POWER CONTROL IN NARROWBAND WIRELESS NETWORKS

In a cellular wireless network, certain quality-of-service (QoS) should be maintained for all active users in the network. A quantity that measures the user’s provided QoS is the signal-to-interference ratio (SIR). In narrowband systems, it is also called carrier-to-interference ratio (CIR). A general idea to achieve the desired SIR for all the active users is to allocate the network resources in the most efficient way. Resource allocation in a cellular wireless network include channel allocation, power control, etc. The transmitter power is the most valuable resource in the network. By proper control of each transmitter power, the interference can be minimized. At the same time, power control extends the battery life in the handset.

In this paper, only narrowband wireless networks are considered. Time and frequency slots are partitioned in a narrowband wireless network in order to avoid mutual interference between users. Nevertheless, users can share the same time or frequency slot provided they are sufficiently far apart, and such users are termed as cochannel users. The issue of power control in narrowband wireless networks arises because the cochannel users interfere with each other.

The optimum open loop power allocation problem is formulated as

$$\min_{p_i} \sum_i p_i, \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (1)

subject to the constraints

$$\gamma_i \geq \gamma_i^{tar}, \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (2)

where $\gamma_i$ is the SIR of user $i$, $\gamma_i^{tar}$ is the desired SIR of user $i$, $p_i$ and $\bar{p}_i$ are the lower and upper bounds on transmitted power $p_i$, respectively. This is a typical linear programming problem, and was considered in [1]. The carrier/ interference (C/I) balancing scheme was discussed by Zander [2]-[3] and Gandhi et.al. [4]-[5]. These schemes achieved C/I balancing, which yields a ‘fair’ distribution of the interference in the sense that all users experience the same C/I level, or have the same CIR/SIR. In [2], a stepwise cell removal and C/I balancing scheme was introduced to achieve minimum outage probability, which is defined by the probability that some randomly chosen mobile has a C/I (CIR/SIR) below a required C/I (CIR/SIR). Proposition 1 in [2] stated that C/I balancing is equivalent to maximization of the achievable CIR/SIR for all users. Proposition 2 in [2] stated that if the CIR/SIR is not achievable, then in order to minimize the outage probability, stepwise cell removal is required to reduce interference.

In [4], the problem of maximizing the minimum CIR/SIR was proposed and was proved to be equivalent to C/I balancing. When the required CIR/SIR is achievable, the problem is reduced to solving a group of linear equations (using the same notation as in [4])

$$(I - \gamma A) P = 0$$ \hspace{1cm} (4)

where $A = G - I$, $G$ is the normalized link gain matrix, $\gamma$ is the maximum achievable CIR/SIR, and $P$ is the corresponding power vector. The above equation implies

$$AP = \frac{1}{\gamma} P$$ \hspace{1cm} (5)

Hence $\frac{1}{\gamma}$ is the eigenvalue of $A$ and $P$ is the corresponding eigenvector. Note that $P$ is not unique. Furthermore, $A$ has zeros at the diagonal and very small positive numbers at the off-diagonal, which means the eigenvalues of $A$ are also very small numbers.

The work of Foschini [7] presented dynamics of CIR/SIR evolution (using the same notation as in [7]) by

$$\dot{p}_i(t) = -\beta (p_i(t) - \rho), \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (6)

where $p_i$ is the CIR/SIR of the $i$ th user, $\rho$ is the desired CIR/SIR, and $\beta$ is a positive parameter. Then by assuming the $i$ th user evolve $p_i$ as if the interference were not going to change, the key differential equation of power evolution is obtained in [7] by taking ‘a surrogate derivative’ of CIR/SIR

$$\dot{p}_i(t) = -\beta (p_i(t) - (\rho/a_i + \sum_{j \neq i} a_{ij} p_j(t))) , \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (7)
where $\nu$ is the background noise, $a_{ij}$ is the link gain from the $j$th mobile to the $i$th base. The above equations in matrix form are given by

$$\dot{p}(t) = -\beta B p(t) + \beta \eta$$

(8)

where

$$B = \begin{bmatrix} 1 & -\rho a_{12} & \cdots & -\rho a_{1j} \\ -\rho a_{21} & 1 & \cdots & -\rho a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho a_{ji} & -\rho a_{jj} & \cdots & 1 \end{bmatrix}$$

(9)

$$\eta = \rho [(\nu/a_{11}), (\nu/a_{22}), \cdots, (\nu/a_{jj})]$$

(10)

A proposition which established the convergence of (8) to its steady state value was given in [7]. To prove this proposition, the eigenvalues of $-B$ have to have negative real parts. Since the entries in matrix $B$ depend on the target CIR/SIR and link gains, certain feasibility conditions are needed to guarantee that ‘all the eigenvalues of $-B$ have negative real parts’.

The distributed power control algorithm by Foschini [7] was shown to converge either synchronously [7] or asynchronously [8]. The basic model in [7] is relaxed in [8] to allow asynchronism. Propagation delay is also considered in [8].

There are many ways to improve system capacity through combined techniques. Combined base station assignment and power control technique was proposed in [9]. Instead of using fixed channel allocation, a dynamic channel allocation scheme is employed together with power control to get a better performance.

II. Feasibility Condition for SIR-based Power Control

In this section, we will discuss the feasibility conditions for SIR-based power control in narrowband wireless systems, e.g., TDMA. Furthermore, we will investigate the implication of the feasibility conditions on system capacity. Its practical use on call admission control (CAC) is introduced.

Throughout this section we will study a large, but finite, narrowband cellular radio system consisting of $Q$ cells. Without loss of generality, we start by considering the signal-to-interference ratio (SIR) in the uplink (mobile-to-base) path. The SIR at base $i$, $\gamma_i$, can be expressed as

$$\gamma_i = \frac{g_{i}p_{i}}{\sum_{j \neq i} g_{j}p_{j} + \nu_{i}} , \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (11)

where $p_{i}$ is the power transmitted by mobile $i$, $g_{i}$ is the link gain from the $j$th mobile to the $i$th base, and $\nu_{i}$ is the receiver noise power at the $i$th base.

In this paper we consider a general case where each link has its own SIR target value which is determined by quality of service (QoS) requirements. The performance objective is to guarantee that the SIR of each link is not below some prespecified value, $\gamma_{i}^{\text{tar}}$ for the $i$th link,

$$\gamma_i \geq \gamma_{i}^{\text{tar}} , \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (13)

In [6] it was stated that these SIR target values should be chosen carefully. However, no result or suggestion is given about how to choose the SIR target values. The effect of them on feasibility of the power control problem (13) needs further investigation. In this paper, we derive necessary and sufficient conditions on the existence of feasible solutions of the power control problem defined by (11) and (13), which depend on the chosen SIR target values and the link conditions (link gains).

A necessary and sufficient condition for $Q = 2$ is stated in the following theorem.

Theorem 1 (Noiseless case for $Q = 2$, $\nu_i = 0$, $i = 1, 2$)

Under the assumption that $p_i, g_{ij}, \gamma_{i}^{\text{tar}}$, $i, j = 1, 2$ are all positive, the necessary and sufficient condition for the existence of feasible solutions $p_1$ and $p_2$ of the power control problem (13) is

$$\gamma_{1}^{\text{tar}}, \gamma_{2}^{\text{tar}} \leq \frac{g_{1}g_{22}}{g_{1}g_{21}}$$

(14)

Formula (14) at the same time provides an upper bound for the product of the SIR target values.

All proofs of the theorems in this paper are given in the appendix.

A necessary and sufficient condition for $Q = 3$ is given in the following theorem.

Theorem 2 (Noiseless case)

Assume that $p_i, g_{ij}, \gamma_{i}^{\text{tar}}$, $i = 1, 2, 3; j = 1, 2, 3$ are all positive. A necessary condition that a feasible solution exists for the power control problem (13) for $Q = 3$ is

$$\gamma_{1}^{\text{tar}}, \gamma_{2}^{\text{tar}} \leq \frac{g_{1}g_{21}}{g_{1}g_{22}} \frac{g_{2}g_{31}}{g_{2}g_{32}} \frac{g_{3}g_{11}}{g_{3}g_{12}}$$

(15)

for all $(i, j)$, $i \neq j$ pairs. The sufficient condition is given by

$$(g_{1}g_{22} - \gamma_{1}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{21}g_{22}) (g_{1}g_{23} - \gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{23}) \geq (\gamma_{1}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{1}g_{21} + \gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{23}) (\gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{21} + \gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{23})$$

(16)

$$(g_{1}g_{22} - \gamma_{1}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{21}g_{22}) (g_{2}g_{31} - \gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{31}) \geq (\gamma_{1}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{1}g_{21} + \gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{23}) (\gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{21} + \gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{23})$$

(17)

$$(g_{1}g_{32} - \gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{32}) (g_{2}g_{31} - \gamma_{2}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{2}g_{31}) \geq (\gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{21} + \gamma_{1}^{\text{tar}} \gamma_{3}^{\text{tar}} g_{1}g_{23}) (\gamma_{3}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{3}g_{21} + \gamma_{3}^{\text{tar}} \gamma_{2}^{\text{tar}} g_{3}g_{23})$$

(18)

Note that (16)-(18) is a very restrictive condition unless some additional assumptions are imposed on the order of the coefficients.

The above results for $Q = 2, 3$ can be easily proved true for the noisy case, where the receiver noise is taken into consideration. Actually, since the receiver noise is much smaller than other quantities in the SIR formula, it will not affect the inequalities.

In multiple cells ($Q > 3$) case, a necessary condition is proposed for both noiseless and noisy case.

Figure 1: Radio link in narrowband wireless networks(TDMA)

Earlier works on power control either try to maximize the minimum SIR [2]-[5] or to maintain SIR of all links not below a prefixed target [6]-[8].

$$\gamma_i \geq \gamma_{i}^{\text{tar}} , \quad i = 1, 2, \cdots, Q.$$  \hspace{1cm} (12)
Theorem 3 (Noiseless case, \( \nu_i = 0, \ i = 1,2, \ldots, Q. \))

Under the assumption that \( p_i, g_{ij}, \gamma_i^{\text{tar}}, \ i, j = 1,2, \ldots, Q. \) are all positive, the necessary condition for the existence of feasible solutions of the power control problem (13) is

\[
\gamma_i^{\text{tar}} \gamma_j^{\text{tar}} \leq \frac{g_{ij} g_{ji}}{g_{ii} g_{jj}}, \quad i, j = 1,2,\ldots, Q. \tag{19}
\]

for any \( i \neq j \). Formula (19) provides a very conservative upper bound for the product of SIRs’ target values.

Theorem 4 (Noisy case)

Assume that \( p_i, g_{ij}, \gamma_i^{\text{tar}} \) are all positive for all \( i, j \). The necessary condition for the existence of a solution of the power control problem (13) is

\[
\gamma_i^{\text{tar}} \gamma_j^{\text{tar}} \leq \frac{g_{ij} g_{ji}}{g_{ii} g_{jj}}, \quad i, j = 1,2,\ldots, Q. \tag{20}
\]

for any \( i \neq j \). Formula (20) provides a very conservative upper bound for the product of SIRs’ target values. Note that

\[
\gamma_i^{\text{tar}} \leq \sqrt{\frac{g_{ij} g_{ji}}{g_{ii} g_{jj}}} \tag{21}
\]

when ‘fairness’ issue is raised for all users that belong to the same service class. SIR is an appropriate measure of the user’s QoS. To guarantee that all users receive the same QoS, it is required that \( \gamma_i^{\text{tar}} = \gamma_j^{\text{tar}}, (i = 1,2,\ldots, Q) \).

Consider the following simple numerical example taken from [10]. The gain matrix \( G \) is given by

\[
G = \begin{bmatrix}
0.3288 & 0.0534 \\
0.0602 & 0.3826
\end{bmatrix} \tag{22}
\]

The background noise level is 0.1. According to the above theorem, the maximum feasible SIR target (assuming that \( \gamma_1^{\text{tar}} = \gamma_2^{\text{tar}} \)) is 7.9627 dB. The power vector is \( \mathbf{p} = [3.1657, 3.0235]^T \). In [10], the SIR target was set at 6 dB, which is feasible. Any SIR target bigger than 7.9627 dB is not feasible. For example, a SIR target of 10 dB will result in a negative power vector \( \mathbf{p} = [-4.6844, -4.7570]^T \).

It was pointed out in [14] similar conditions for the achievable SIR when \( Q = 2 \) by considering the determinant of the matrix \( I - \gamma^{\text{tar}} Z \), where matrix \( Z \) is defined in (29). This approach of solving the optimal power allocation in the sense of minimizing power consumption by solving group of linear equations was discussed by Mitra [8], and stated later in (32) and (33) in the next section of this paper. However, [14] only gave the condition of solvability of (32)(33) rather than considering the original power control problem (13).

Using the following definition of a strongly-diagonally-dominant matrix

\[
g_{ii} \gg \sum_{j \neq i} g_{ij} \quad \forall i, j. \tag{23}
\]

We can give additional interpretations of Theorem 4. The necessary condition of Theorem 4 reveals the strongly-diagonally-dominant nature of the link gain matrix \( G = \{g_{ij}\} \) when feasible solutions exist. This is indeed the practical scenario in narrowband wireless systems, e.g., TDMA systems. Note that \( \gamma_i^{\text{tar}} \) are relatively large numbers. Furthermore, if the link gain matrix \( G = \{g_{ij}\} \) is strongly-diagonally-dominant, then there exist infinite many solutions of the SIR-based power control problem. It is easy to verify this by comparing the order of quantities in the following inequality derived from (11) and (13)

\[
p_i \geq \gamma_i^{\text{tar}} \sum_{j \neq i} \frac{g_{ij} p_j}{g_{ii} g_{jj}} + \gamma_i^{\text{tar}} \frac{\nu_i}{g_{ii}}, \ \forall i, j. \tag{24}
\]

where in practical narrowband systems, \( p_i = O(1), \ g_{ij} = O(10^{-3}), g_{ii} \) is between \( O(10^{-3}) \) and \( O(10^{-5}) \) which depends on the strongly-diagonally-dominant nature of the link gain matrix and the frequency reuse factor, the background noise level is negligible, and usually \( \gamma^{\text{tar}} = O(10^2) \), which is less than 20 dB. It is obvious that the above inequality is satisfied if the link gain matrix is strongly-diagonally-dominant. Hence, under the above assumptions about the order of the quantities that define the power control problem (11)(13), we have the following theorem:

Theorem 5 Assuming that \( p_i = O(1), \gamma_i^{\text{tar}} = O(10^2) \), the link gain matrix is strongly-diagonally-dominant with at least \( g_{ij} = O(10^{-3}) g_{ii} \) and with background noise level being negligible, then the power control problem is feasible. There exist infinite many solutions (\( p_i, i = 1,2,\ldots, Q. \)) satisfying (13) so that the above assumption provides the sufficient condition for the existence of feasible solutions of the power control problem defined by (11) and (13).

Theorem 4 has immediate implication on system capacity. It provides an insight of how cochannel interference will affect system capacity. Cochannel interference cannot be combated by simply increasing the transmitter power. The maximum achievable SIR of simultaneously active users using the same frequency band or the same time slot depends on link (channel) gains.

Furthermore, if all the cells have similar fading conditions, Theorem 4 gives the quantitative relation between the maximum achievable SIR (of simultaneously active users using the same frequency band or the same time slot) and the reuse factor.

Theorem 4 gives a necessary condition for a solution to exist of the power control problem, and at the same time, a bound on the achievable SIR. We could get a better bound by considering some practical issues in existing narrowband systems, e.g., the frequency reuse pattern in TDMA systems. A TDMA system with reuse factor 1/7 is shown in Figure 2, where \( f1, \ldots, f7 \) denote different frequency bands.

Figure 2: TDMA System with frequency reuse factor 1/7
Assume that all cochannel users transmit with similar powers, i.e., $p_i \approx p_j$. Then for noiseless case,
\[
\gamma_{\text{tar}} = \frac{g_i}{\sum_{j \neq i} g_j}
\]  
\[
(25)
\]
A widely used simple model of link gain is given in [11], namely, $g_{ij} = d_{ij}^{-\gamma} A_{ij}$, where $d_{ij}$ is the distance between the $j$th mobile to the $i$th base station, $A_{ij}$ is contributed by shadow fading. Assume that the fading in all cells are similar, i.e., $A_{ij}$ are similar for all $i$ and $j$. Let the number of nearest cochannel cells be $M - 1$, where $1/M$ is the frequency reuse factor. When all cells have the same size
\[
\gamma_{\text{tar}} \leq \frac{1}{M-1} \left( \frac{d_{ij}}{d_{ii}} \right)^{4}
\]  
\[
(26)
\]
for any $i \neq j$. The above result is consistent with the result in [12], which stated that the cochannel interference ratio is independent of the transmitted power and is a function of the radius of the cell and the distance between centers of the nearest cochannel cells, when all the cells have the same size. The result obtained in (26) represents a sharper bound for the SIR target value than the value from the necessary condition. A similar upper bound was derived for CDMA systems in [13].

In Figure 2, let $CD = \frac{2r}{\sqrt{\pi}}$, where $r$ is the radius of the cell, then it is easy to calculate that the distance between centers of the nearest cochannel cells $AE = 2\sqrt{\pi}CD = \sqrt{21}r$. It is easy to derive the relation between the maximum achievable SIR (of simultaneously active users using the same frequency band or the same time slot) and the reuse factor. For example, if the reuse factor is $1/7$, in the worst case (when mobiles locate at the border of the cells)
\[
\frac{d_{ij}}{d_{ii}} = \frac{d_{ij}}{d_{ij}} = \frac{\sqrt{21} - \sqrt{39/48}}{\sqrt{39/48}}
\]  
\[
(27)
\]
\[
\gamma_{\text{tar}} \leq \frac{1}{6} \left( \frac{\sqrt{21} - \sqrt{39/48}}{\sqrt{39/48}} \right)^{4} = 16.7 \text{ dB}
\]  
\[
(28)
\]
Similar SIR value was reported in [12].

Another application of the feasibility conditions is call admission control (CAC). As pointed out in [2], in a mobile system, the link gains will constantly change. It will be difficult to verify the above feasibility conditions because usually the link gains are unknown. Even if it is possible, it would require a significant measurement effort. In addition, the amount of measured data and signalling between bases and mobiles would be enormous in a reasonably sized network. However, based on a propagation model or empirical data of link gain measurements, precomputed acceptable range of SIR threshold can be obtained and stored in a database at the control center. The database contains table of acceptable range of SIR threshold vs. number of simultaneous users in each cell. The databases could be used as references for call admission control (CAC).

III. IMPROVED STEPWISE REMOVAL ALGORITHM

In this section, we show how to improve Zander’s stepwise removal algorithm (SRA) in [2] by using the results obtained in the previous section.

Let $Z$ be the following nonnegative matrix
\[
Z_{ij} = \begin{cases} 
\frac{g_i}{g_j} & i \neq j \\
0 & i = j 
\end{cases}
\]  
\[
(29)
\]
Let $\lambda^*$ be the largest real eigenvalue of matrix $Z$ and $p^*$ be the corresponding eigenvector. Let $\gamma^* = \frac{1}{\lambda^*}$ be the maximum achievable C/I ratio $\gamma_0$. The stepwise removal algorithm (SRA) proposed by Zander in [2] is listed below.

**Algorithm 1**

**Step 1.** Determine $\gamma^*$ corresponding to $Z$. If $\gamma^* \geq \gamma_0$, use the eigenvector $p^*$, else set $Q = Q$.

**Step 2.** Remove the cell $k$ for which the maximum of the row and column sums is maximized and form the $(Q-1) \times (Q-1)$ matrix $Z'$. Determine $\gamma^*$ corresponding to $Z'$. If $\gamma^* \geq \gamma_0$, use the eigenvector $p^*$, else set $Q = Q - 1$ and repeat Step 2.

By removing the cells that cause too much interference to other cells, the outage probability is minimized while achieving a SIR level higher than the SIR threshold for all remaining users. Based on Theorem 5, it is inappropriate to remove the cell which has the maximum row and column sums. Instead, the cell $k$ or $l$ which satisfy
\[
Z_{kl}Z_{lk} = \max_{i \neq j} \{Z_{ij}Z_{ji}\}, \quad i, j = 1, 2, ..., Q.
\]  
\[
(30)
\]
should be removed. Note that $Z_{ij}Z_{ji} = \frac{g_i g_j}{g_0 g_{ij}}$.

Consider the following numerical example.

\[
Z = 10^{-4} \begin{bmatrix}
0 & 2.8 & 8.7 & 2.2 & 10^{-2} & 10^{-2} & 10^{-2} & 4.2 \\
3 & 0 & 0 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} \\
5.5 & 10^{-2} & 0 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} \\
1.2 & 10^{-2} & 10^{-2} & 0 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} \\
3.3 & 10^{-2} & 10^{-2} & 10^{-2} & 0 & 10^{-2} & 10^{-2} & 10^{-2} \\
2.3 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 0 & 10^{-2} & 10^{-2} \\
2.1 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 0 & 10^{-2} \\
10 & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 10^{-2} & 0
\end{bmatrix}
\]
\[
(31)
\]
It is obvious that $Z_{34}Z_{45}$ is the maximum product, however, SRA will remove cell 1 based on the row and column sums. After cell 1 is removed, the remaining cells can not achieve a better performance than before due to $\gamma_{\text{tar}} < \sqrt{\frac{1}{Z_{34}Z_{45}}}$.

Assume that all the links have the same SIR target value $\gamma_{\text{tar}}$. If a feasible solution exists, then there exists a unique solution which minimize the transmitter power in a Pareto sense. This solution is obtained by solving a system of linear equations, as stated in Proposition 2.1 in [8]. The Pareto optimal solution is
\[
p^* = \left[ I - \gamma_{\text{tar}} Z \right]^{-1} u
\]  
\[
(32)
\]
where $u$ is the vector with elements
\[
u_i = \gamma_{\text{tar}} \frac{g_i}{g_{ii}}, \quad i = 1, 2, ..., Q
\]  
\[
(33)
\]
Suppose the matrix $Z$ has a pair of entries $Z_{kl}$ and $Z_{lk}$ which satisfy (30). We propose the following Improved Stepwise Removal Algorithm (ISRA)
**Algorithm 2**

*Step 1.*
Determine the power allocation by using the formula (32). If it is feasible, then all the transmitting powers are positive, and use $p$ as the power vector. Else go to Step 2.

*Step 2.*
Find cell $k$ and $l$ which satisfy (30). Then remove cell $k$ or $l$, which has larger row and column sums. Form the remaining link gain matrix, remove the corresponding term in $u$ and go back to Step 1.

IV. CONCLUSIONS

Based on the above discussion, we draw the following conclusion of the SIR-based power control problem in narrowband systems: If the link gain matrix is strongly-dominantly-dominant with the realistic values of the problem quantities, and the desired SIR is chosen properly, then there exist infinite many solutions of the SIR-based power control problem. Otherwise, there is hardly a solution since the existence conditions are very restrictive.

The feasibility conditions given in Section 2 have immediate implication on the capacity of narrowband systems. They provide an insight of how cochannel interference will affect system capacity. The effect of cochannel interference on the SIR of cochannel users can not be decreased by simply increasing the transmitted power. The maximum achievable SIR of simultaneously active users using the same frequency band or the same time slot depends on link (channel) gains.

According to the necessary and sufficient conditions derived in Section 2, an improved stepwise removal algorithm (ISRA) is introduced to remove the cell which cause the worst interference to other cells. The achievable SIR is increased for all remaining cells by applying ISRA.

V. APPENDIX

Proof of Theorem 1.
The power control problem (13) for $Q = 2$ requires that

$g_{11}p_1 - \gamma_{\text{tar}}^{1}g_{12}p_2 \geq 0$

$g_{22}p_2 - \gamma_{\text{tar}}^{2}g_{21}p_1 \geq 0$

which under assumptions that $g_{ij}$ and $\gamma_{\text{tar}}^{i}$ are positive implies

$\frac{g_{22}p_2}{\gamma_{\text{tar}}^{2}g_{21}} \geq p_1 \geq \frac{\gamma_{\text{tar}}^{1}g_{12}p_2}{g_{11}}$

$\frac{g_{11}p_1}{\gamma_{\text{tar}}^{1}g_{12}} \geq p_2 \geq \frac{\gamma_{\text{tar}}^{2}g_{21}p_1}{g_{22}}$

These two parts of inequalities are satisfied when

$\frac{g_{22}p_2}{\gamma_{\text{tar}}^{2}g_{21}} \geq \frac{\gamma_{\text{tar}}^{1}g_{12}p_2}{g_{11}}$

$\frac{g_{11}p_1}{\gamma_{\text{tar}}^{1}g_{12}} \geq \frac{\gamma_{\text{tar}}^{2}g_{21}p_1}{g_{22}}$

Assuming that $p_i > 0$, $i = 1, 2$, the result stated in (14) follows. In such a case any $p_1$ and $p_2$ that in the ranges

$\frac{g_{22}p_2}{\gamma_{\text{tar}}^{2}g_{21}} \geq p_1 \geq \frac{\gamma_{\text{tar}}^{1}g_{12}p_2}{g_{11}}$

$\frac{g_{11}p_1}{\gamma_{\text{tar}}^{1}g_{12}} \geq p_2 \geq \frac{\gamma_{\text{tar}}^{2}g_{21}p_1}{g_{22}}$

are feasible solutions for the power control problem (13). The obtained result is both necessary and sufficient for the existence of feasible solutions.

Note that if we allow that $p_i \geq 0$, $i = 1, 2$, then $p_1 = 0$ implies $p_2 = 0$, and vice versa, so that we have trivial solution $\gamma_1 = \gamma_2 = 0$. This requires that $p_i$, $i = 1, 2$, must be strictly positive for $Q = 2$.

Proof of Theorem 2.
The feasibility condition is

$g_{11}p_1 - \gamma_{\text{tar}}^{1}g_{12}p_2 - \gamma_{\text{tar}}^{2}g_{21}p_1 - \gamma_{\text{tar}}^{3}g_{31}p_2 \geq 0$

$g_{22}p_2 - \gamma_{\text{tar}}^{2}g_{21}p_1 - \gamma_{\text{tar}}^{3}g_{32}p_1 \geq 0$

$g_{33}p_3 - \gamma_{\text{tar}}^{3}g_{31}p_2 - \gamma_{\text{tar}}^{3}g_{32}p_1 \geq 0$

These inequalities imply that

$g_{22}p_2 - \gamma_{\text{tar}}^{2}g_{21}p_1 - \gamma_{\text{tar}}^{3}g_{32}p_1 \geq 0$

$g_{33}p_3 - \gamma_{\text{tar}}^{3}g_{31}p_2 - \gamma_{\text{tar}}^{3}g_{32}p_1 \geq 0$

$g_{11}p_1 - \gamma_{\text{tar}}^{1}g_{12}p_2 - \gamma_{\text{tar}}^{3}g_{31}p_2 \geq 0$

By grouping two inequalities together, we get the sufficient condition. The necessary condition can be derived from the three inequalities (17)-(19) easily due to the positivity assumption on all coefficients and variables.

Proof of Theorem 3.
From (11) and (13) we have

$g_{1i}p_i \geq \gamma_{i}^{\text{tar}}(\sum_{k \neq i} g_{ik}p_k)$

$g_{ij}p_j \geq \gamma_{j}^{\text{tar}}(\sum_{k \neq j} g_{kj}p_k)$

Combining these two inequalities we obtain

$g_{ij}p_j - \gamma_{j}^{\text{tar}}(\sum_{k \neq i} g_{kj}p_k) \geq p_i \geq \gamma_{i}^{\text{tar}}(\sum_{k \neq i} g_{ik}p_k)$

which can be written in the form

$(g_{ij}g_{jj} - \gamma_{j}^{\text{tar}}\gamma_{i}^{\text{tar}}g_{ij}g_{ji})p_j \geq \gamma_{i}^{\text{tar}}g_{ii}(\sum_{k \neq i} g_{ik}p_k)$

$+ \gamma_{i}^{\text{tar}}\gamma_{j}^{\text{tar}}g_{ji}(\sum_{k \neq i,j} g_{ik}p_k)$

Since the right-hand side is positive due to the assumptions, it follows that

$g_{ij}g_{jj} - \gamma_{j}^{\text{tar}}\gamma_{i}^{\text{tar}}g_{ij}g_{ji} \geq 0$

must hold for any $p_j$, $j = 1, 2, \cdots, Q$. The last set of inequalities represent the necessary condition stated in Theorem 3.
Proof of Theorem 4.
The following two inequalities should be satisfied for any $i \neq j$.

\[ g_{ii}p_i \geq \gamma^\text{tar}_i \left( \sum_{k \neq i} g_{ik}p_k \right) + \gamma^\text{tar}_i \nu_i \]

\[ g_{jj}p_j \geq \gamma^\text{tar}_j \left( \sum_{k \neq j} g_{jk}p_k \right) + \gamma^\text{tar}_j \nu_j \]

The above two inequalities imply that

\[
g_{jj}p_j - \frac{g^\text{tar}_j}{g_{ii}} \sum_{k \neq i} g_{jk}p_k - \gamma^\text{tar}_j \nu_j \\
\geq \frac{\gamma^\text{tar}_i \sum_{k \neq i} g_{ik}p_k + \nu_i}{g_{ii}}
\]

This is equivalent to

\[
(g_{ii}g_{jj} - \gamma^\text{tar}_i \gamma^\text{tar}_j g_{ij}g_{ji})p_j \geq \gamma^\text{tar}_j \gamma^\text{tar}_i \left( \sum_{k \neq i,j} g_{jk}p_k + \nu_j \right) \\
+ \gamma^\text{tar}_i \gamma^\text{tar}_j \gamma^\text{tar}_j \left( \sum_{k \neq i,j} g_{ik}p_k + \nu_i \right)
\]

Since the right-hand side is positive due to the assumptions, it follows that

\[ g_{ii}g_{jj} - \gamma^\text{tar}_i \gamma^\text{tar}_j g_{ij}g_{ji} \geq 0 \]

must hold for any $p_j$, $j = 1, 2, \cdots, Q$. The last set of inequalities represent the necessary condition stated in Theorem 4.

REFERENCES


