Optimal Distributed Power Control in Cellular Wireless Systems

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Abstract

In this paper, an optimal fast closed-loop SIR-based power control scheme is proposed for CDMA systems. The state-of-the-art of this power control algorithm is that the power evolution is directly driven by the error between the actual SIR and the desired SIR. The amount of power change is proportional to the error, and the optimal gain is chosen to achieve the minimum error at the next time instant. Assuming perfect estimation and prediction, the error between the desired SIR and the real value of SIR goes to zero in one step. In practical situations, where the estimator need several (3-4) steps to make accurate estimation and prediction, the error between the desired SIR and the real value of SIR goes to zero in several (4-5) steps. The proposed controller is an estimator based controller. The performance of the estimator will have direct impact on the performance of the power control scheme. Simulations of a CDMA system are performed to demonstrate effectiveness of the proposed power control scheme. The simulation results, in all cases, confirm theoretical analysis. These results indicate that the proposed optimal power control scheme is a very promising solution to the distributed fast closed-loop power control problem in wireless systems, both theoretically and practically.

I. Review of SIR based Power Control in Cellular Wireless Systems

In a cellular wireless network, certain quality-of-service (QoS) should be maintained for all active users in the network. A quantity that measures the user’s provided QoS is the signal-to-interference ratio (SIR). The initial work on SIR-based power control schemes for narrowband systems has been done by Zander [1]-[3].

Power control schemes can be centralized [1]-[3] or distributed [2],[4],[6] according to the nature of computations. A centralized controller has information (e.g., all the link gains) for each user and it decides control actions for all users. On the other hand, a distributed controller only uses local information to make a control action for a user. For example, [2],[4],[6] only use information about user’s own link gain and/or SIR to make a decision about their transmitted power.

The algorithm convergence has to be studied carefully when a distributed power control scheme is used. The distributed power control algorithm by Foschini [6] was shown to converge either synchronously [6] or asynchronously [9]. The basic model in [6] is relaxed in [9] to allow asynchronism. Propagation delay is also considered in [9]. A framework for convergence of the generalized uplink power control was proposed by Yates [10].

In recent years, there have been increasing interests in applying code-division-multiple-access (CDMA) techniques to cellular wireless networks. Unlike the narrowband systems, users in a CDMA system share the same frequency all the time by using a specific spread spectrum pseudonoise code for each user. The system performance mainly depends on so-called multiple access interference (MAI), which is caused by the cross-correlations between the desired signal and other signals.

Open-loop power control has been employed to combat the path loss and shadow fading. The average power control in [10],[22] keeps the local mean received power to a constant value, which mitigate the effect of shadowing and near-far problems. In IS-95B [20] and CDMA2000 [21], fast closed-loop power control is proposed to fight medium to fast fading.
In [11]-[12], it is shown that when power control rate is higher, it can partially accommodate the effect of fast fading.

Herdtner and Chong [9] considered a simple distributed power control scheme in which the convergence condition was given. Wu [13] extended Zander’s problem formulation to CDMA systems by reordering the number of users in different cells. Song et.al. [14]-[15] and Gunnarsson et.al. [16]-[17] considered up/down power control and closed-loop power control with other nonlinear elements within the framework of nonlinear control systems. [14]-[15] gave guidelines for choosing the appropriate power control step size in IS-95. In [16]-[17], PID controllers were designed to overcome the effects of time-delay in the feedback loop.

Filtering techniques have been also used to design power control schemes. Leung [18] proposed a power control scheme for TDMA data service based on the Kalman filtering technique. In [31], a linear prediction of received power is made at the base station to predict the power control bit one step ahead. Both approaches assume that the interference is Gaussian.

In this paper, an optimal fast closed-loop SIR-based uplink power control scheme is proposed for CDMA systems. In Section 2, a proportional controller is proposed and the optimal power control gain is derived by unconstrained optimization. In Section 3, the constraints are imposed on the maximum and minimum values of transmitter power. By applying the Kuhn-Tucker conditions, we derive the optimal gain for the proportional controller. Using Bellman’s optimality principle, the power control scheme achieves optimality in the whole time interval of interest. The proposed controller is an estimator based controller. The performance of the estimator will have direct impact on the performance of the power control scheme. Section 4 gives our choice of the estimator. In contrast to the well-known Kalman filter, which assumes that system and measurement disturbances are white Gaussian stochastic processes and requires the knowledge of their statistics, the $H_\infty$ filter, used in the proposed scheme, facilitates the system filtering regardless of the nature of the system and measurement disturbances. It meets our estimation requirement since the fluctuation of the interference is not Gaussian and its statistics is not known in general [18]. Simulations of a CDMA system are performed to demonstrate the effectiveness of the proposed power control scheme. The results are shown in Section 5.

II. Optimal SIR-based Distributed Power Control by Unconstrained Optimization

In this paper, only uplink (mobile-to-base) power control is considered. Suppose that there are $N$ active users in the system. The Signal-to-Interference Ratio (SIR) in a CDMA system is defined as

Definition 1: The SIR of an active link from mobile station $i$ to base station $n$ in a wireless system is defined as

$$\gamma_{ni} = \frac{Lh_{ni}p_i}{\sum_{j\neq i} h_{nj}p_j + \sigma^2}$$

(1)

where $h_{ni}$ is the link gain from mobile station $i$ to base station $n$. We assume that $L$ is the processing gain in spread spectrum wireless systems, e.g., in CDMA 2000, $L=64$ or 128 or 256. Let us denote the denominator by $I_i$, which represents the received interference, then the SIR can be rewritten as

$$\gamma_{ni} = \frac{Lh_{ni}p_i}{I_i}$$

(2)

Assuming that a mobile only transmits to one base station during the time of power control, then $\gamma_{ni}$ can be simplified as $\gamma_i$. Let $\delta_i = \frac{h_{ni}}{I_i}$ denote the channel variation [11]. $\delta_i$ will be estimated and predicted in the proposed power control scheme.
Given that a mobile station and its base station communicate directly to each other, we assume that \( h_{ni} \) can be estimated accurately. We also assume that the received interference \( I_i \) can be measured at each time step. The received interference is the difference between the total received power and the desired signal power [10].

The main idea of the proposed algorithm is to make the transmission power proportional to the error between the actual SIR and the desired SIR. Define the transmission power change from time step \( k \) to \( k+1 \) as

\[
\Delta p_i(k+1) = p_i(k+1) - p_i(k)
\]  

where \( p_i(k) \) and \( p_i(k+1) \) are the transmission powers from mobile \( i \) to the base station at the time instants \( k \) and \( k+1 \), respectively. Let the error between the actual SIR (\( \gamma_i(k) \)) and the desired SIR (\( \gamma_{i\text{tar}} \)) be denoted by \( e_i(k) \) for mobile \( i \) at the time instant \( k \),

\[
e_i(k) = \gamma_{i\text{tar}} - \gamma_i(k)
\]

We propose the following power control algorithm

\[
\Delta p_i(k+1) = \alpha_i(k)e_i(k) \implies p_i(k+1) = p_i(k) + \alpha_i(k)(\gamma_{i\text{tar}} - \gamma_i(k))
\]

where \( \alpha_i(k) \) is the gain to be determined through the optimization procedure in which the square of the error is minimized (theoretically reduced to zero in each discrete-time instant \( k \)). The main result of this paper is presented in the following theorem.

**Theorem 1:** Define the performance criterion at the time instant \( k \) as

\[
J_i(k) = \min_{\alpha_i(k)}(e_i(k+1))^2, \; i = 1, 2, ..., N.
\]

The optimal gains of the power control algorithm (5) that minimize \( J_i(k) \) at every time instant \( k \) are given by

\[
e_{i\text{opt}}(k) = \begin{cases} 
\frac{1}{\delta_i(k)}(1 - \frac{\gamma_{i\text{tar}}}{e_i(k)}) + \frac{1}{\delta_i(k+1)}\frac{\gamma_{i\text{tar}}}{e_i(k)} & \text{if } e_i(k) \neq 0 \\
0 & \text{if } e_i(k) = 0
\end{cases}
\]

where \( \delta_i(k) = \frac{Lh_{ni}(k)}{I_i(k)} > 0 \), \( \delta_i(k+1) = \frac{Lh_{ni}(k+1)}{I_i(k+1)} > 0 \) and \( h_{ni}(k), h_{ni}(k+1), I_i(k), I_i(k+1) > 0, \forall k \).

The minimal value of the performance criterion is zero when the optimal gain \( \alpha_{i\text{opt}}(k) \) is applied.

**Proof:** Firstly, we derive the error dynamic equation using (1)-(5)

\[
e_i(k+1) = (-\delta_i(k+1)\alpha_i(k) + \frac{\delta_i(k+1)}{\delta_i(k)}e_i(k))(1 - \frac{\delta_i(k+1)}{\delta_i(k)}\gamma_{i\text{tar}})
\]

Secondly, we compute the square of the error

\[
(e_i(k+1))^2 = (\alpha_i(k)+\frac{\delta_i(k+1)}{\delta_i(k)}e_i(k))(1 - \frac{\delta_i(k+1)}{\delta_i(k)}\gamma_{i\text{tar}})^2 + 2(-\delta_i(k+1)\alpha_i(k) + \frac{\delta_i(k+1)}{\delta_i(k)}e_i(k))(1 - \frac{\delta_i(k+1)}{\delta_i(k)}\gamma_{i\text{tar}})
\]

From the necessary conditions for optimality at the time instant \( k \)

\[
\frac{\partial J_i(k)}{\partial \alpha_i(k)} = \frac{\partial (e_i(k+1))^2}{\partial \alpha_i(k)} = 0
\]
we have

\[ \alpha^\text{opt}_i(k) = \frac{1}{\delta_i(k)}(1 - \frac{\gamma_{\text{tar}}^i}{e_i(k)}) + \frac{1}{\delta_i(k+1)} \frac{\gamma_{\text{tar}}^i}{e_i(k)} \]  

(11)

Taking the second derivative we obtain

\[ \frac{\partial^2 J_i(k)}{\partial \alpha_i^2(k)} = 2(\delta_i(k+1)e_i(k))^2 > 0 \]  

(12)

which indicates that the resulted \( J_i(k) \) by using \( \alpha_{\text{opt}}^i(k) \) is at minimum and establishes the sufficiency part of the theorem. Another obvious proof of sufficiency follows from the fact that \( J_i(k) \geq 0 \) and \( J_i(\alpha^\text{opt}_i(k)) = 0 \). Using Bellman’s optimality principle [29], we achieve optimality in the whole time interval of interest by applying this power control algorithm at each time step. In other words, the controller gains obtained in (7) are optimal for all \( k \). ■

The block diagram of the proposed distributed power control algorithm is shown in Figure 1. The state-of-the-art of this power control algorithm is that the power evolution is directly driven by the error between SIR and the desired SIR. The amount of power change is proportional to the error, and the gain is chosen to achieve the minimum error at the next time instant \( k + 1 \).

In traditional CDMA systems, e.g., IS-95, power control was used to combat path loss and shadow fading. Rayleigh fading was compensated by advanced modulation and coding schemes. On the other hand, in 3-G wireless systems, e.g., CDMA-2000, UMTS, the closed-loop power control is faster than before. For example, in IMT-2000 proposal [21], the frequency of closed-loop power control is 1600 Hz. It should be fast enough to compensate Rayleigh fading. Traditional approaches using power iterations or the implemented up/down power control schemes are not appropriate to compensate Rayleigh fading, because both schemes requires rather long time to converge. Usually, they assume that the link gains and fading conditions are not changed much during the whole power control period.

An important assumption behind all the power iterative algorithms reviewed in Section 1 is that power updates occur so quickly that the link gains can be assumed to be fixed. Our power control algorithm is not an iterative algorithm. The assumption that the link gains do not change during power evolution is not required in the proposed algorithm. On the other hand, the proposed power control algorithm is suitable for such purpose due to the immediate decision of the optimal power allocation of the next time step based on measurements and predictions. With a good estimator (for example, discrete-time \( H_\infty \) filter), all the effects including path loss, shadow fading and Rayleigh fading, will be taken into consideration. The error between the actual SIR and the desired SIR is driven to zero (theoretically) in every single step.

The block diagram of the proposed power control scheme with an estimator is shown in Figure 2, where \( f() \) is defined as

\[ f(x_1, x_2) = \begin{cases} 
\frac{1}{x_1}(1 - \frac{\gamma_{\text{tar}}^i}{e_i(k)}) + \frac{1}{x_2} \frac{\gamma_{\text{tar}}^i}{e_i(k)} & \text{if} \quad e_i(k) \neq 0 \\
0 & \text{if} \quad e_i(k) = 0 
\end{cases} \]  

(13)
Figure 2. The optimal distributed power control system with an estimator $f_1(x) = 10\log_{10}(x)$ and $f_2(x) = 10^{x/10}$ are converting functions between $W$ and $dBW$.

It is noticeable that the proposed controller is an estimator based controller. The idea of quick online estimation of link quality was mentioned in [19]. It pointed out that any adaptive power control scheme need quick detection of the change of link quality. The performance of the estimator will have a direct impact on the performance of the power control scheme. Let $\hat{\delta}_i(k)$ and $\hat{\delta}_i(k+1)$ denote the estimated/predicted value of $\delta_i(k)$ and $\delta_i(k+1)$, respectively. Then the gain applied to the power control algorithm is

$$\hat{\alpha}_i(k) = \begin{cases} \frac{1}{\delta_i(k)}(1 - \frac{\gamma_{tar}}{\delta_i(k)}) + \frac{1}{\delta_i(k+1)} e_i(k) & \text{if } e_i(k) \neq 0 \\ 0 & \text{if } e_i(k) = 0 \end{cases}$$

The error between the actual SIR and the desired SIR at the next time instant $k+1$ is

$$e_i(k+1) = \delta_i(k+1)[(\frac{1}{\delta_i(k)} - \frac{1}{\delta_i(k+1)})e_i(k) - \gamma_{tar} + (\frac{1}{\delta_i(k+1)} - \frac{1}{\delta_i(k+1)})\gamma_{tar}]$$

$$= p_i(k+1)(\hat{\delta}_i(k+1) - \delta_i(k+1))$$

It is obvious that if the estimated/predicted values of $\delta_i(k)$ and $\delta_i(k+1)$ are close to the real values, then $e_i(k+1) \approx 0$. The power evolution is given by

$$p_i(k+1) = p_i(k) + \hat{\alpha}_i(k)e_i(k) = \frac{\gamma_{tar}}{\delta_i(k+1)}$$

III. OPTIMAL SIR-BASED DISTRIBUTED POWER CONTROL BY CONSTRAINED OPTIMIZATION

In the previous section, we have performed unconstrained optimization to get the optimal gain for the controller. However, in real wireless systems, there are always constrains on the power that the mobile can transmit, namely, the highest transmitted power allowed and the lowest transmitted power to maintain connections. In this section, we will derive the optimal gain for our controller by using constrained optimization.

Suppose that the highest transmitted power allowed is $p^{max}$ and that the lowest transmitted power allowed is $p^{min}$. We formulate the following constrained optimization problem

$$J_i(k) = \min_{\alpha_i(k)}(e_i(k+1))^2$$
subject to

\[ 0 < p^{\text{min}} \leq p_i(k), \quad p_i(k+1) \leq p^{\text{max}} \]  \hspace{1cm} (18)

The solution of the above nonlinear programming problem is given by

\[
p_i(k+1) = \begin{cases} 
p^{\text{min}} & \text{if } \delta_i(k+1) > \frac{\gamma^{\text{tar}}}{p^{\text{min}}} \\
p^{\text{max}} & \text{if } \delta_i(k+1) < \frac{\gamma^{\text{tar}}}{p^{\text{max}}} \\
p_i(k) + \alpha_i(k)e_i(k) & \text{o.w.}
\end{cases}
\]  \hspace{1cm} (19)

where \( \alpha_i(k) \) was defined in (7). The detailed derivation of the result in (19), obtained by using the well-known Kuhn-Tucker conditions [28], can be found in [32].

From equation (19), we draw the following conclusion: Under normal operations, the optimal power allocation hardly hits the lower/upper bound. They can only happen under extreme conditions, namely, when the desired SIR is either set too low \( \delta_i(k+1)p^{\text{min}} > \gamma^{\text{tar}} \) or too high \( \delta_i(k+1)p^{\text{max}} < \gamma^{\text{tar}} \). These extreme conditions can happen in the proposed power control scheme when \( \gamma_i^{\text{tar}} \) is not set properly and/or when measurements and/or predictions are far from the real values. These extreme conditions provide a mean to evaluate the quality of the \( H_\infty \) filter (or any other estimator to be used) when \( \gamma_i^{\text{tar}} \) is set properly. It can be stated as

\[ \text{if } \hat{\delta}_i(k+1) < \frac{\gamma^{\text{tar}}}{p^{\text{max}}}, \text{ or if } \hat{\delta}_i(k+1) > \frac{\gamma^{\text{tar}}}{p^{\text{min}}}, \text{ then the predictions are bad.} \]

However, we should keep in mind that the performance of the \( H_\infty \) filter does not depend on the value of \( \gamma_i^{\text{tar}} \). It mainly depends on the choice of the optimization parameters of the \( H_\infty \) filter. These parameters will be introduced in Section 4.

Based on the above discussions, the modified power control algorithm, which can be implemented in existing wireless systems, is proposed as follows:

\[
p_i(k+1) = \begin{cases} 
p^{\text{min}} & \text{if } \hat{\delta}_i(k+1) > \frac{\gamma^{\text{tar}}}{p^{\text{min}}} \\
p^{\text{max}} & \text{if } \hat{\delta}_i(k+1) < \frac{\gamma^{\text{tar}}}{p^{\text{max}}} \\
p_i(k) + \hat{\alpha}_i(k)e_i(k) & \text{o.w.}
\end{cases}
\]  \hspace{1cm} (20)

where \( \hat{\alpha}_i(k) \) is defined in (14). Formula (20) is consistent with the Pontriagin’s minimum principle which stated that in constrained optimization, optimal solutions lie on the boundary [29].

The block diagram of the complete power control system composed of \( N \) links with estimators can be found in [32], where the effect of the correlation in the received power from interfering users is taken into account.

In order to compute \( \hat{\alpha}_i(k) \), we need a ‘good’ estimator/predictor. The \( H_\infty \) filter is a very promising candidate. We will discuss this issue in the next section.

IV. \( H_\infty \) Filter as an Optimal Estimator/Predictor

The problem of filtering involves estimation of system states using past measurements. The development of efficient linear estimators, e.g., the Kalman filter, has been based mainly on the minimization of the \( L_2 \)-norm of the corresponding estimation error. This type of estimation assumes that the message generating process has a known dynamics and that the exogenous inputs have known statistical properties. The well-known Kalman filter offers optimal filtering algorithm when the system model parameters are known and the system and measurement noise is represented by white Gaussian processes with known statistics (power spectral density). In [18], Leung applied the Kalman filter to predict the interference in the immediate future. Its usage is constrained by the requirement of the Kalman filter,
i.e., the fluctuation of the interference has to be Gaussian white noise stochastic process, which is not the case in general [18].

In contrast to Kalman filtering, the $H_\infty$ filter [23]-[27] considers the system filtering regardless of the nature of the system and measurement disturbances, as long as the power of the disturbance is bounded [26]-[27].

In the proposed power control scheme, $\delta(k)$ and $\delta(k+1)$ have to be estimated and predicted, respectively. The fluctuations of $\delta(k)$ and $\delta(k+1)$ do not have normal distributions in general. Hence, the Kalman filter is not appropriate for the problem under consideration. On the other hand, the $H_\infty$ filter does not require any knowledge of the statistics of system and measurement disturbances. This makes the $H_\infty$ filter an appropriate estimator for the proposed power control scheme.

The dynamics of $\delta_i$ (in dB) is described by

$$
\delta_i(k+1) = \delta_i(k) + w_i(k)
$$

where $w_i(k)$ represents the process noise (disturbance). Let $y_i(k)$ be the measurement of $\delta_i(k)$.

$$
y_i(k) = \delta_i(k) + v_i(k)
$$

where $v_i(k)$ is the measurement noise. Due to the scalar nature of the problem, it is obvious that the system controllability and observability conditions are met [33]. These conditions are needed for the existence of the appropriate solution of the algebraic Riccati equation [34].

The $H_\infty$ filter (suboptimal) solves the following optimization problem

$$
\sup_{Q_i, W_i, V_i} \frac{\sum_{k=0}^{N-1} ||\delta_i(k) - \hat{\delta}_i(k)||_{Q_i}^2 + \sum_{k=0}^{N-1} ||w(k)||_{W_i}^2 + ||v(k)||_{V_i}^2}{\lambda} < 1 \tag{23}
$$

where $Q_i, W_i, V_i$ are positive weighting parameters chosen by the designer, and $\lambda$ is a prescribed level of noise attenuation [26]-[27]. The discrete-time $H_\infty$ filter is given by

$$
\hat{\delta}_i(k+1) = \hat{\delta}_i(k) + K_i(k)(y_i(k) - \hat{\delta}_i(k)) \tag{24}
$$

where $K_i(k)$ is the optimal gain of the $H_\infty$ filter. It is computed from the following equation

$$
K_i(k) = P_i(k)(I - \lambda Q_i P_i(k) + V_i^{-1}(k) P_i(k))^{-1} V_i^{-1}(k) \tag{25}
$$

Since in our problem $P_i(k), Q_i$ and $V_i$ are all scalars, the gain can be found as

$$
K_i(k) = \frac{P_i(k)}{(1 - \lambda Q_i P_i(k) + \frac{P_i(k)}{V_i}) V_i} \tag{26}
$$

and $P_i(k)$ is the unique positive solution of the following scalar difference Riccati equation

$$
P_i(k+1) = P_i(k)(I - \lambda Q_i P_i(k) + V_i^{-1}(k) P_i(k))^{-1} + W_i \tag{27}
$$

Again, since $P_i(k), Q_i, W_i$ and $V_i$ are all scalars, (27) can be written as

$$
P_i(k+1) = \frac{P_i(k)}{1 - \lambda Q_i P_i(k) + \frac{P_i(k)}{V_i} + W_i} \tag{28}$$
The initial condition of the system is \( \delta_i(0) = \delta_{i0} \). The initial condition of the above \( H_\infty \) filter is \( P_i(0) = P_{i0} \). These initial conditions are arbitrary positive quantities. It can be shown that the steady-state value of \( P_i(k) \) satisfies

\[
P_{i,ss} = \frac{W_i(\frac{1}{V_i} - \lambda Q_i) \pm \sqrt{(W_i(\frac{1}{V_i} - \lambda Q_i))^2 + 4W_i(\frac{1}{V_i} - \lambda Q_i)}}{2(\frac{1}{V_i} - \lambda Q_i)} \quad (29)
\]

Only the positive one is the steady-state solution of \( P_i(k) \). It is obvious that the choice of the optimization parameters \( \lambda, Q_i, W_i \) and \( V_i \) has an impact on the filter performance. The block diagram of our \( H_\infty \) filter is shown in Figure 3.

![Figure 3. The structure of the discrete-time \( H_\infty \) filter](image)

The performance of this discrete-time \( H_\infty \) filter will be tested and discussed in the next section.

V. Simulation Results and Analysis

In this section, simulations of a CDMA system are performed to demonstrate effectiveness of the proposed power control scheme. Comparison between the proposed optimal power control scheme and other existing power control schemes clearly shows the advantages of the proposed scheme.

A CDMA system with 7 hexagonal cells and 16 users per cell is considered. The operating frequency is 1.9 GHz and the bandwidth of each channel is assumed to be 1.23 MHz, which is in accordance to the CDMA 2000 standard [21]. Data rate is set at 9600 bps and the processing gain is set at 128 (21 dB). The target SIR is 7 dB, which corresponds to the bit error rate (BER) being less than \( 10^{-3} \). Note that the SIR, as defined in this paper, includes the processing gain, which is denoted by \( \frac{E_b}{N_0} \) in the standard IS-95B [20]/CDMA 2000 [21], where \( E_b \) is the energy per information bit and \( N_0 \) is the interference power spectral density.

In simulation we further make the following assumptions:

1. The effects of antenna directivity, voice activity factor, and soft handoff are ignored.
2. The minimum and maximum power can be transmitted by a mobile (IS-95B[20]) are \( p_{min} = 8 \) dBm (6.3 mW) and \( p_{max} = 33 \) dBm (2 W), respectively.
3. The background noise power is 0.05 mW.
4. The transmitted power is updated periodically, every 0.625 msec, which corresponds to 1,600 Hz fast closed-loop power control frequency proposed in IMT-2000.
5. The antenna of the base station and all mobiles are omnidirectional.
6. Mobiles are assumed to be uniformly distributed in a cell.
7. It is assumed that the link gains have the following form

\[
h_{ni}(k) = d_{ni}^{-4}(k)A_{ni}(k)B_{ni}(k) \quad (30)
\]

where \( d_{ni}(k) \) is the distance from the \( i \)th mobile to the \( n \)th base station at time instant \( k \), \( A_{ni} \) is a log-normal distributed stochastic process, and \( B_{ni} \) is contributed by Rayleigh fading [30].
8. It is assumed that the cell diameter is 2 km. $d_{m}(k)$ is a 2-D uniformly distributed random variable, [30].

9. It is assumed that the standard deviation of $A_{ni}$ is 8, [30].

10. The speed of the mobiles are ranging from 0.5 mph to 60 mph. The maximum Doppler frequency is 80 Hz, which correspond to 60 mph mobile speed. $B_{ni}(k)$ is simulated using a filtered Gaussian noise model, [30].

11. The initial mobile transmitted power is taken as $p_{0} = p_{\text{min}} + \eta(p_{\text{max}} - p_{\text{min}})$, where $\eta \in [0, 1]$ is a uniformly distributed random number.

12. The mean values of the link gains are used in the simulation since only the deterministic case is considered in this paper.

The optimization parameters of the $H_{\infty}$ filter are $\lambda = 0.05$, $Q_{i} = 20$, $W_{i} = 1$ and $V_{i} = 0.1$ for all mobiles. In addition, $\delta_{0}$ and $P_{0}$ are positive random numbers using random number generators.

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Figure 4. Comparison between the optimal power control scheme, DCPC and IS-95:

(a). Power evolution

(b). Error performance

We consider three cases, based on the range of the desired SIR, namely, when $\gamma_{\text{tar}}$ is feasible, when $\gamma_{\text{tar}}$ is set too high, and when $\gamma_{\text{tar}}$ is set too low. The simulation results can be found in [32]. In all three cases the simulation results fit theoretical analysis (Section 2) very well. These results indicate that the optimal power control scheme is a very promising solution of the distributed fast closed-loop power control problem in wireless systems, both theoretically and practically.

In the current IS-95 systems, the power control scheme is up/down hard decision type [20]. It can not achieve zero SIR error, due to persistent oscillations around zero [14].

A widely known and accepted power control scheme is called the Distributed Constrained Power Control (DCPC), and is given by [5]

$$p_{i}(k + 1) = \min\{\frac{\gamma_{\text{tar}}}{\gamma_{i}(k)}p_{i}(k), \ p_{\text{max}}\} \quad (31)$$

The performances of the proposed optimal power control scheme, DCPC, and IS-95 are compared in Figure 4. When the channel is stationary (the mean values of the channels are constants), the optimal scheme will save at least 2 frames, that would be lost if the other two power control schemes were used.
When channel changes frequently, the estimator used will have good tracking of the channel quality and the proposed optimal power control scheme will also provide accurate values for power adjustments almost instantly. On the other hand, DCPC and up/down scheme in IS-95 will not converge fast enough and will not provide appropriate power changes.

REFERENCES

4(a) Power evolution of user 1

4(b) Error performance of user 1