Letter

Wireless Systems

Optimal linear and bilinear algorithms for power control in 3G wireless CDMA networks

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SUMMARY

In this paper we present two power control algorithms that potentially can be used as efficient techniques for power updates in 3G wireless CDMA communication networks. The algorithms are obtained by optimising the signal-to-interference-ratio (SIR) error. The control laws are distributed, respectively linear and bilinear, and either requires estimation of the channel interference (or the quantity inversely proportional to the signal interference, known as the channel variation) or the development of efficient numerical algorithms for solving a set of obtained algebraic equations. Simulation results show superiority of the proposed algorithm over the distributed constrained power control (DCPC) algorithm. Copyright © 2006 AEIT.

1. INTRODUCTION

The classic work on dynamic control of transmitted power in spread spectrum mobile cellular radio systems is due to References [1–3]. The authors based their results on the works of References [4,5], on a similar problem in satellite communications. These approaches were based on maximisation of the Signal-to-interference ratio (SIR), see also References [6] and [7], where the average power control techniques were proposed to mitigate both near-far effects and the effect of shadowing.

It is commonly accepted that the modern approach SIR-based power control for wireless systems originated in the works of Zander [8,9] and his coworkers [10–12] as well as in the often cited paper by Foschini and Miljanic [13]. The algorithms of References [9] and [13] require only the local information (the user’s own SIR information) to calculate the local user’s power. For that reason, they are called the distributed algorithms. The distributed power control algorithm [13] was shown to converge either synchronously or asynchronously [15]. A framework for convergence of the generalised uplink power control was proposed in Reference [14], see also Reference [16]. Reference [17] shows that mobile’s outage probability can be dramatically decreased whenever the minimum required SIR is not achieved, and suggested dropping one of mobiles, specifically the mobile whose removal most improves the remaining SIRs. A distributed power

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control algorithm with active link protection has been studied in Reference [18]. In Reference [19], a numerical linear algebra technique based on the use of the successive over-relaxation technique [20] is proposed to speed up distributed power control algorithms.

In the IS-95B and CDMA2000 standards, fast closed-loop power control is proposed to fight medium to fast fading. In References [21,22], it is shown that a higher power control rate can partially accommodate the effect of fast fading. Several closed-loop power algorithms proposed in Reference [23] have the ability to compensate for the time-varying channel characteristics. Reference [24] presented a simple asynchronous distributed power control scheme based on the schemes used in IS-95 with the corresponding convergence condition. References [25,26] and [27,28] have considered up/down power control and closed-loop power control with other nonlinear elements within the framework of nonlinear control systems. Reference [25,26] gave guidelines for choosing the appropriate power control step size in IS-95. In Reference [27], PID controllers were designed to overcome the effects of time-delay in the feedback loop. Reference [29] has formulated and solved the power updates problem in CDMA networks using a Nash game theoretic approach.

The paper [30] studied the stochastic power control problem formulation using match filters and the Gaussian white noise assumption. The idea of quick online estimation of link quality was initiated in Reference [31]. Reference [32] proposed a power control scheme for TDMA data service based on the Kalman filtering technique. The Kalman filter is used for integrated power control and adaptive modulation coding in wireless packet-switched networks in Reference [33]. In Reference [34], a linear prediction of received power is made at the base station to predict the power control bit one step ahead. Both approaches assume that the interference is Gaussian. In References [35,36], an optimisation approach using an estimator has been employed to solve the stochastic mobile power update problem in wireless CDMA systems, with no assumption imposed on the stochastic nature of the interference. Reference [37] studied the stochastic power control problem under Gaussian white noise in the SIR measurements via a joint power and SIR error variance minimisation technique.

1.1. Iterative methods for SIR-based power control in wireless networks

Consider the uplink (mobile-to-base) power control problem with \( N \) active users in a CDMA wireless system. For such a system, the SIR is defined by

\[
\gamma_i = \frac{g_{ii}p_i}{\sum_{j=1,j\neq i}^{n} g_{ij}p_j + \sigma_i^2} = \frac{g_{ii}p_i}{I_i(p_{-i})}, \quad i = 1, 2, \ldots, n \tag{1}
\]

where \( p_i \) is the transmission power for user \( i \), \( g_{ij} \) is the link gain from mobile station \( j \) to base station \( i \) and \( \sigma_i \) is receiver noise (background noise) at base station \( i \). \( I_i(p_{-i}) \) in Equation (1) denotes the interference power for mobile \( i \), where \( p_{-i} \) indicates that the power of mobile \( i \) is not present in the interference it experiences.

The goal in the power control of wireless systems is that every mobile has SIR above a certain target value, that is

\[
\gamma_i \geq \gamma_{i,\text{tar}}, \quad i = 1, 2, \ldots, n \tag{2}
\]

Assuming equalities in Equation (2) and knowledge of all gains \( g_{ij} \), Equation (1) represents a system of linear algebraic equations of the form

\[
A \mathbf{p} = \mathbf{b}, \quad \mathbf{p} = [p_1, p_2, \ldots, p_n]^T \tag{3}
\]

with the elements of \( A \) and \( \mathbf{b} \) given by \( a_{ij} = -\gamma_{i,\text{tar}}^{-1} g_{ij}, i \neq j, a_{ii} = 1, b_i = \sigma_i^2/\gamma_{i,\text{tar}}^{-1} g_{ii} \). This system can be directly solved for \( p_i, i = 1, 2, \ldots, n \), using Gaussian elimination. However, in reality (in IS-95 and CDMA 2000 power control mechanism) mobile \( i \) knows only its own SIR, \( \gamma_i(k) \), at discrete-time instants, \( k = 0, 1, 2, \ldots \).

The paper [13] suggested solving Equation (3) using the following distributed algorithm

\[
p_i(k+1) = \frac{\gamma_{i,\text{tar}}}{\gamma_i(k)} p_i(k), \quad i = 1, 2, \ldots, n \tag{4}
\]

where

\[
\gamma_i(k) = \frac{g_{ii}p_i(k)}{\sum_{j=1,j\neq i}^{n} g_{ij}p_j(k) + \sigma_i^2} = \frac{g_{ii}p_i(k)}{I_i(p_{-i}(k))} = \delta_i(k)p_i(k) \tag{5}
\]

The quantity \( \delta_i(k) \) is called the channel variation, [21,22]. According to Equation (5), the channel variation is defined by

\[
\delta_i(k) = \frac{g_{ii}p_i(k)}{I_i(p_{-i}(k))} = \frac{\gamma_i(k)}{p_i(k)} \tag{6}
\]

Note that in currently implemented IS-95 power updates scheme, the signal \( \gamma_i(k) \) is measured on line. It can be shown that Equation (4) represents Jacobi iterations [20] for solving algebraic Equations (3). A nice feature of the

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The algorithm presented in Equation (4) is that information about the current mobile’s power and the current SIR is sufficient to update the mobile’s power. The algorithm is called the distributed power control (DPC) algorithm. In Reference [38], the DPC algorithm is combined with the algorithm of Reference [39] via a soft computing technique. In Reference [40], an adaptive self-tuning power algorithm is proposed with the DPC algorithm serving as a reference one.

In the next section, we present two new algorithms based on optimisation of the error between the SIR actual and target values. These algorithms are along the lines of [8,9] and [13,14] and they are motivated by our previous results reported in Reference [29] and [36]. The first algorithm, in fact represents the linear discrete-time controlled system, and the second algorithm is the discrete-time bilinear controlled system (linear in both the state and control variables, but nonlinear due to the product of state and control variables). These algorithms are applicable to CDMA 2000 wireless networks [41]. Corresponding CDMA 2000 closed-loop control structure is discussed in detail in our papers [29,37,42].

2. PROPOSED ALGORITHMS

Our motivation for the first algorithm comes from the recent power control results for wireless networks reported in References [29] and [36].

Algorithm 1: (Linear and additive power updates)

We propose that the mobile power can be updated according to the following distributed linear control law

\[ p_i(k + 1) = p_i(k) + u_i(k), \quad i = 1, 2, \ldots, n \]  

(7)

where the control variable \( u_i(k) \) has to be chosen such that the square of the SIR error, defined by

\[ e_i(k) = \gamma_i^{\text{tar}} - \gamma_i(k), \quad i = 1, 2, \ldots, n \]  

(8)

is minimised at the discrete time instant \( k + 1 \), that is

\[ \min_{u_i(k)} \{ e_i^2(k + 1) \} \]  

(9)

Since a mobile has control of its own power only, and has no control of power (and interference) of other mobiles that use the same communication channel, the mobile can only take the partial derivative of its own performance criterion (9) with respect to its own control signal defined in Equation (7). This is, in fact equivalent to the necessary condition for

\[ \partial(e_i(k + 1))^2 = \partial(\gamma_i^{\text{tar}} - \gamma_i(k + 1))^2 \]

\[ = \partial(\gamma_i^{\text{tar}} - \gamma_i(k + 1)p_i(k + 1))^2 \]

\[ = \partial(\gamma_i^{\text{tar}} - \gamma_i(k + 1)(p_i(k) + u_i(k)))^2 \]

\[ = -2\delta_i(k + 1)(\gamma_i^{\text{tar}} - \gamma_i(k + 1)(p_i(k) + u_i(k))) = 0 \]

which leads to the following optimal solution (note that all quantities in the above optimisation procedure implicitly depend on \( u_i(k) \), however, this implicit dependence does not need to be included in the optimisation process [29,36])

\[ u_i^*(k) = \gamma_i^{\text{tar}} - \gamma_i(k) \quad i = 1, 2, \ldots, n \]  

(10)

with

\[ \delta_i^*(k + 1) = \frac{g_{ii}p_i^*(k + 1)}{I_i(p_i^*(k + 1))} = \frac{\gamma_i^*(k + 1)}{p_i^*(k + 1)} = \frac{\gamma_i^{\text{tar}}}{p_i^*(k + 1)} \]  

(11)

The optimal solution given in terms of the channel interference is

\[ u_i^*(k) = \frac{1}{g_{ii}}(\gamma_i^{\text{tar}}I_i(p_i^*(k + 1) - \gamma_i^*(k))I_i(p_i^*(k))) \]

(12)

The corresponding optimised controlled power updates are given by

\[ p_i^*(k + 1) = p_i^*(k) + u_i^*(k) \]

\[ = \frac{\gamma_i^{\text{tar}}I_i(p_i^*(k + 1))}{g_{ii}}, \quad i = 1, 2, \ldots, n \]  

(13)

or using Equation (5)

\[ p_i^*(k) = \frac{\gamma_i^{\text{tar}}I_i(p_i^*(k))}{\delta_i^*(k)} = \frac{\gamma_i^{\text{tar}}}{\delta_i^*(k)} \quad i = 1, 2, \ldots, n \]  

(14)

Note that since

\[ \{ e_i^*(k + 1) \}^2 = (\gamma_i^{\text{tar}} - \gamma_i(k + 1)(p_i^*(k) + u_i^*(k)))^2 \]

\[ = (\gamma_i^{\text{tar}} - \gamma_i(k + 1) + \delta_i(k + 1) - \delta_i(k))p_i^*(k) + u_i^*(k) = 0 \]
the result obtained in Equation (10) also represents the sufficient condition for minimisation of the defined cost function $e_i^2(k+1) \geq 0$.

It should be emphasised that the error cost can be theoretically reduced instantaneously to zero assuming that the channel interference (or channel variation) is exactly known. In practise, an estimator (observer) is needed to estimate the channel interference, which will require several iterations (power updates) before the error settles down to zero. The use of an estimator to estimate the channel variation is considered in References [35,36]. In practise, using an estimator, the optimal power updates obtained in Equation (14) could be implemented as

$$p_i^*(k) = \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k_i^*)^T (p_{i,\text{est}}^*(k)), \quad i = 1, 2, \ldots, n \quad (15)$$

or

$$p_i^*(k) = \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k_i^*)^T, \quad i = 1, 2, \ldots, n \quad (16)$$

where $k_{\text{est}}^*(p_{i,\text{est}}^*(k))$ and $\delta_{\text{est}}(k)$ are, respectively, the estimated values for the channel interference and the channel variation. The accuracy of the estimates of the channel interference or channel variation directly determine the accuracy of the optimal powers. However, since in practise the gains and interference change (in IS-95 the powers are updated 800 times per second), we should not aim for the exact optimal solution. Using a near-optimal solution at the given discrete-time instant is acceptable since we have to move very rapidly (hopefully not at the speed of 800 Hz) to the next time instant and generate a new estimate of the channel interference (channel variation).

Comment: In view of the result obtained in Equation (14), we can notice that the fixed point algorithm for solving algebraic Equations (14) leads to the DPC algorithm defined in Equation (4), which indicates that in such a case the solution of Equation (14) is identical to the power balancing solution, or more precisely, that the power balancing solution is optimal in the sense that it brings the SIR error to zero (in general, after a lot of iterations)—It is known that the fixed point algorithms have slow convergence. The research is underway to find more efficient algorithms for solving Equation (14). However, in practical implementation of Equation (14), we have a one shot solution defined by Equation (15) or (16), depending whether an estimate of the channel interference or the channel variation is known. Note that in the wireless communications literature, several papers study the problem of interference, estimation: Reference [36]—where $H_{\infty}$ filter was used to estimate and predict channel variation, [43]—where the Kalman filter was used for interference estimation, see also References [32,34] and [44,45].

In summary of Algorithm 1, to spell out the difference between the proposed algorithm and the DPC algorithm, the DPC algorithm basically solves Equation (14) via the fixed point iterations and hence calculates the ‘expected (estimated)’ interference, which leads to Equation (4). In contrast, the proposed algorithm is based on the use of an interference estimator. If the estimator is perfect, we practically have one shot solution. In practise, no estimator is perfect so that several iterations are needed to obtain the sought solution.

Algorithm 2: (Bilinear control law)

The mobile power can also be updated according to the following distributive bilinear control law

$$p_i(k+1) = p_i(k)u_i(k), \quad i = 1, 2, \ldots, n \quad (17)$$

where $u_i(k)$ is the control variable. Using the same cost function as the one defined in Equation (9), the necessary conditions imply the following solution

$$u_i^*(k) = \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k)$$

The corresponding optimised controlled power updates are given by

$$p_i^*(k+1) = p_i^*(k)u_i^*(k)$$

$$= \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k) (p_{i,\text{est}}^*(k+1)), \quad i = 1, 2, \ldots, n \quad (19)$$

or using Equation (8)

$$p_i^*(k) = \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k) (p_{i,\text{est}}^*(k)) = \frac{\delta_{\text{tar}}}{\delta_{\text{est}}}(k), \quad i = 1, 2, \ldots, n \quad (20)$$

which is identical to Equation (14)—the result obtained for the linear (additive) power updates. Hence, both linear and bilinear control laws produce the same optimal power update algorithm. This result is interesting from both theoretical and practical points of view. It is theoretically known that we can achieve more with multiplicative control than with additive control. However, in our problem formulation and corresponding derivations, it has happened that both the linear and bilinear algorithms produce the same power updates. Practically, the additive control law (7) is much simple for implementation than the multiplicative control law (17).
In this section, simulations of a CDMA system are performed to demonstrate effectiveness of the proposed power control scheme. A comparison between the proposed optimal power control scheme and the distributed constrained power control algorithm (DCPC), [12], clearly shows the advantages of the proposed scheme. Note that the DCPC algorithm has been considered in the wireless communication literature as the most efficient power control algorithm and almost exclusively used as a comparison test for new algorithms.

A CDMA system with 7 hexagonal cells and 16 users per cell is considered. The operating frequency is 1.9 GHz and the bandwidth of each channel is assumed to be 1.23 MHz, which is in accordance to the CDMA 2000 standard [41]. Data rate is set at 9600 bps and the processing gain is set at 128 (21 dB). The target SIR is 7 dB, which corresponds to the bit error rate (BER) being less than $10^{-3}$. Note that the SIR, as defined in this paper, includes the processing gain, which is denoted by $\frac{E_b}{N_0}$ in the standard IS-95B/CDMA 2000 [41], where $E_b$ is the energy per information bit and $N_0$ is the interference power spectral density.

In simulation we further make the following assumptions:

1. The effects of antenna directivity, voice activity factor and soft handoff are ignored.
2. The minimum and maximum power can be transmitted by a mobile (IS-95B [46]) are $p_{\text{min}} = 8$ dBm (6.3 mW) and $p_{\text{max}} = 33$ dBm (2 W), respectively.
3. The background noise power is 0.05 mW.
4. The transmitted power is updated periodically, every 0.625 ms, which corresponds to 1600 Hz fast closed-loop power control frequency.
5. Mobiles are assumed to be uniformly distributed in a cell.
6. Cell diameter is 2000 m.
7. It is assumed that the link gain is composed of path loss and log-normal shadowing, where the order of path loss is 4 and the standard deviation of shadowing is 8 dB.
8. The initial mobile transmitted power is taken as $p_0 = p_{\text{min}} + \eta(p_{\text{max}} - p_{\text{min}})$, where $\eta \in [0, 1]$ is a uniformly distributed random number.

An $H_\infty$ filter is adopted as the optimal estimator in the proposed power control scheme. The details of the $H_\infty$ filter can be found in References [35] and [36].
words, the estimation error is reduced by more than 6 orders of magnitude in five steps. As the estimation error goes to zero, the mobile transmission power also converges very fast. In about 10 steps, all the SIRs converge to the desired SIR values (Figure 1).

A widely known and accepted power control scheme is called the DCPC, and is given by Reference [12]

\[
p_i(k + 1) = \min \left\{ \frac{\gamma_i^{\text{tar}}}{\gamma_i(k)} p_i(k), p_{i}^{\text{max}} \right\}
\]

(21)

It is derived from Equation (4) taking into the account that the mobile power can not exceed its maximal value.

In the second experiment, the performances of the proposed optimal power control scheme and DCPC are compared in Figure 3.

It can be observed from this figure that the proposed algorithm has fast convergence. The DCPC algorithm is slow and falls behind. Another advantage of the proposed scheme is that it will adapt to channel changes quickly since it is an estimator based controller. This property is obvious when channel quality has abrupt change, as in Figure 3 (channel quality becomes worse at power control step 11). When channel changes frequently, the estimator used will have good tracking of the channel quality and the proposed optimal power control scheme will also provide accurate values for power adjustments almost instantly. On the other hand, DCPC will not converge fast enough and can not keep pace with the channel changes.

4. CONCLUSIONS

We have derived the optimal additive (linear) and multiplicative (bilinear) power update laws and shown that they lead to the same algorithm. The linear power update control law is more convenient for implementation due to its simplicity. Further studies are needed to find a numerically efficient algorithm for solving obtained set of algebraic Equations (14), unless an estimator is used to estimate either channel interference or channel variation, in which case formula (16) is an approximate optimal solution (as accurate as the interference estimate). Simulation results performed indicate superiority of the proposed algorithm over the DCPC algorithm.

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