Energy Efficient Resource Allocation in Cognitive Radio Wireless Ad Hoc Networks

Song Gao¹, Lijun Qian¹, and D.R. Vaman²

¹Prairie View A&M University, ²CeBCom Research Center
U.S.A

1. Introduction

Recent technological advances have resulted in the development of wireless ad hoc networks which are envisioned to provide rapid on-demand network deployment due to their self-configurability and lack of pre-deploy infrastructure requirements. These devices generally have small form factors, and have embedded storage, processing and communication ability (I. F. Akyildiz (2009)). With the growing proliferation of such wireless devices, the spectrum is increasingly getting congested. However, it has also been pointed out in several recent measurement reports that the spectrum are highly under-utilized (FCC (2002)). In order to achieve much better spectrum utilization and viable frequency planning, Cognitive Radios (CRs) are under development to dynamically capture the unoccupied spectrum (J. Mitola (1999)). Many challenges arise with such dynamic and hierarchical means of accessing the spectrum, especially for the dynamic resource allocation of CR users by adapting their transmission and reception parameters to the varying spectrum condition while adhering to power constraints and diverse quality of service (QoS) requirements (see, for example, S. Tao (2006); Q. Zhao (2007)). In this chapter, an energy constrained wireless CR ad hoc network is considered, where each node is equipped with CR and has limited battery energy. One of the critical performance measures of such networks is the network lifetime. Additionally, due to the infrastructureless nature of ad hoc networks, distributed resource management scheme is desired to coordinate and maintain communications between each transmitting receiving pair. In this context, the present chapter provides a framework of distributed energy efficient spectrum access and resource allocation in wireless CR ad hoc networks that employ orthogonal frequency division multiple access (OFDMA) (K. Fazel (2003); A. Pandharipande (2002)) at the physical layer. OFDMA is well suited for CR because it is agile in selecting and allocating subcarriers dynamically and it facilitates decoding at the receiving end of each subcarrier (J. Bazerque (2007)). In addition, multi-carrier sensing can be exploited to reduce sensing time (I. F. Akyildiz (2006)). Each emerging CR user will select its subcarriers and determine its transmission parameters individually by solving an optimization problem. The optimization objective is to minimize its energy consumption per bit while satisfying its QoS requirements and power limits.

¹which is defined as the ratio of the total transmission and reception power consumption over available subcarrier set to its achieved throughput.
Compared with the power minimization with respect to target data rate constraints S. Tao (2006) or throughput maximization under power upper bound Q. Zhao (2007), this objective function, which measures the total energy consumed for reliable information bits transmitted, is particularly suitable for energy constrained networks where the network lifetime is a critical metric.

Although the emerging CR users will not cause harmful interference to the existing users, they may choose the same subcarriers in the same time slot independently, and thus co-channel interference may be introduced. In this work, we allow multiple new users to share the same subcarriers as long as their respective Signal-to-Interference-and-Noise-Ratio (SINR) is acceptable. This may be achieved by distributed power control R. Yates (1995), which converges very fast. The flow chart of the proposed distributed energy efficient spectrum access and resource allocation scheme is highlighted in Fig. 1, where step 2 corresponds to the constrained optimization performed by each emerging user individually. More detailed illustrations of the flow chart are given in section 4.

Resource allocation problem in wireless ad hoc networks has been extensively investigated in the literature. In G. Kulkarni (2005), the resource allocation problem is explored for OFDMA-based wireless ad hoc network by directly adopting distributed power control scheme for the power and bits allocation on all subcarriers to improve power efficiency. A greedy algorithm is proposed for best subcarrier selection in CR networks employing multicarrier CDMA Q. Qu (2008), and distributed power control is performed thereafter to resolve co-channel interference. An Asynchronous Distributed Pricing (ADP) scheme is proposed in J. Huang (2006), where the users need to exchange information indicating the interference caused by each user to others. In the context of CR enabled wireless sensor network (WSN) S. Tao (2006), a two-step algorithm is proposed to tackle the allocation problem: channel assignment with objective of minimizing transmission power and channel contention to reserve the subcarrier set for transmission by intended transmitters, while the interference spectrum mask is assumed to be known a priori. The authors of Q. Zhao (2007) address the opportunistic spectrum access (OSA) problem in WSN, in which a distributed channel allocation problem is modeled by a partially observable Markov decision process.
framework (POMDP) while assuming the transition probability of each channel is known. In Y. T. Hou (2007), the CR spectrum sharing problem is formulated in multi-hop networks with objective to minimize the space-bandwidth product (SBP). However, the transmission power allocated on each subcarrier is assumed to be the same which may lead to significant performance loss. The effect of power control is analyzed in a subsequent work Y. Shi (2007). Dynamic Frequency Hopping Community (DFHC) is proposed in W. Hu (2007) for the spectrum sharing in CR based IEEE 802.22 wireless regional area networks (WRANs) to ensure QoS satisfaction and reliable protection to licensed users.

In this chapter, a new constrained optimization problem is formulated and solved that minimizing energy per bit across users subject to QoS and power constraints in a multi-user ad hoc network. A novel concept, “energy-efficient waterfilling”, is given in this section that is fundamentally different from the rate-adaptive waterfilling or margin-adaptive waterfilling. In this case the optimal point is located in the constraint interval rather than on the boundary. In fact, the rate-adaptive and margin-adaptive waterfilling can be considered as special cases of the energy-efficient waterfilling presented in this work. The results obtained provide a valuable insight that the optimal solution of energy efficient resource allocation is not best subcarrier selection for multiple transmitting receiving pairs in an OFDMA network S. Gao (2008). The proposed distributed subcarrier selection and power allocation scheme provides an efficient and practical solution for dynamic spectrum access in CR wireless ad hoc networks employing OFDMA. By combining the optimal resource allocation of individual users and distributed power control, the proposed method guarantees fast convergence speed, computational efficiency and implementation simplicity. Motivated by iterative waterfilling (IWF) algorithm in W. Yu (2002), another distributed solution may be obtained by solving the multi-user distributed channel and power allocation problem iteratively. However, it may take many steps for the iterative algorithm to converge if it converges at all and the delay may be too large to be tolerable. The cost of the additional computation complexity is high. On the contrary, the proposed optimal resource allocation of individual users is easy to obtain and distributed power control algorithm has well-known fast convergence speed. Furthermore, it will be shown that the proposed distributed algorithm performs closely to the global optimal point.

2. System model

We consider an energy constrained CR OFDMA network of $N$ communicating pairs. Both transmitter $i$ and receiver $j$ is indexed by $\mathcal{N} := \{1, 2, ..., N\}$. If $j = i$, receiver $j$ is said to be the intended receiver of transmitter $i$. The transmission system is assumed to be a time-slotted OFDMA system with fixed time slot duration $T_S$. Slot synchronization is assumed to be achieved through a beaconing mechanism. Before each time slot, a guard interval is inserted to achieve synchronization, perform spectrum detection as well as resource allocation (based on the proposed scheme). Inter-carrier interference (ICI) caused by frequency offset of the side lobes pertaining to transmitter $i$ is not considered in this work (which can be mitigated by windowing the OFDM signal in the time domain or adaptively deactivating adjacent subcarriers T. Weiss (2004)).

A frequency selective Rayleigh fading channel is assumed at the physical layer, and the entire spectrum is appropriately divided into $M$ subcarriers to guarantee each subcarrier

---

2The optimal allocation strategy with objective to minimize power or maximize throughput is named margin-adaptive and rate-adaptive waterfilling over frequency channels, respectively.
experiencing flat Rayleigh fading S. Kondo (1996). We label the subcarrier set available to the transmitter receiver pair $i$ after spectrum detection by $\mathcal{L}_i \subset \{1,2,\ldots,M\}$. Let $G := \{G^k_{i,j}, i,j \in \mathcal{N}, k \in \mathcal{L}_i\}$ denote the subcarrier fading coefficient matrix, where $G^k_{i,j}$ stands for the sub-channel coefficient gain from transmitter $i$ to receiver $j$ over subcarrier $k$. $G^k_{i,j} = |H^k_{i,j}(f)|^2$, where $|H^k_{i,j}(f)|$ is the transfer function. It is assumed that $G$ adheres to a block fading channel model which remains invariant over blocks (coherence timeslots) of size $T_s$ and uncorrelated across successive blocks. The noise is assumed to be additive white Gaussian noise (AWGN), with variance $\sigma^2_{i,k}$ over subcarrier $k$ of receiver $i$. We define $P := \{p^k_i, p^k_j \geq 0, i \in \mathcal{N}, k \in \mathcal{L}_i\}$ as the transmission power allocation matrix for all users in $\mathcal{N}$ over the entire available subcarrier set $\cup_{i \in \mathcal{N}} \mathcal{L}_i$, where $p^k_i$ is the power allocated over subcarrier $k$ for transmitter $i$. For each transmitter $i$, the power vector can be formed as

$$p^i := [p^1_i, p^2_i, \ldots, p^M_i]^T$$

(1)

If the $k^{th}$ subcarrier is not available for transmitter $i$, $p^k_i = 0$. Each node is not only energy limited but also has peak power constraint, i.e., $\sum_{k \in \mathcal{L}_i} p^k_i \leq p^\text{max}_i$. The set of all feasible power vector of transmitter $i$ is denoted by $\mathcal{P}_i$

$$\mathcal{P}_i := \left\{ p^i \in \prod_{k \in \mathcal{L}_i} [0, p^\text{max}_i], \sum_{k \in \mathcal{L}_i} p^k_i \leq p^\text{max}_i \right\}$$

(2)

The signal to interference plus noise ratio (SINR) of receiver $i$ over subcarrier $k$ ($\gamma^k_i$) can be expressed as

$$\gamma^k_i(p^k) = \frac{\alpha^k_i(p^k_j) \cdot p^k_i}{\sum_{j \neq i \in \mathcal{N}} G^k_{i,j} \cdot p^k_j + \sigma^2_{i,k}}$$

(3)

where $\alpha^k_i$ is defined as the channel state information (CSI) which treats all interference as background noise. $\alpha^k_i$ can be measured at the receiver side and is assumed to be known by the corresponding transmitter through a reciprocal common control channel.

When all users divide the spectrum in the same fashion without coordination, it is referred to as a Parallel Gaussian Interference Channel which leads to a tractable inner bound to the capacity region of the interference model. The achievable maximum data rate for each user (Shannon’s capacity formula) is

$$c_i(p^i) = \sum_{k \in \mathcal{L}_i} \gamma^k_i(p^k) = \sum_{k \in \mathcal{L}_i, p^k_i \in \mathcal{P}_i} \log_2 \left( 1 + \alpha^k_i(p^k_j) \cdot p^k_i \right)$$

(4)

where $B^k_i$ is the equally divided subcarrier bandwidth for transmitter $i$. Without loss of generality, $B^k_i$ is assumed to be unity in this work. The noise is assumed to be independent of the symbols and has variance $\sigma^2$ for all receivers over entire available subcarrier set. Furthermore, all communicating transmitter and receiver pairs are assumed to have diverse
QoS requirements specified by $\sum_{k \in L_i} c_i^k \geq r_i^{tar}$, where $r_i^{tar}$ is the target data rate of transmitter $i$.

In an energy constrained network (such as a wireless sensor network), reception power is not negligible since it is generally comparable to the transmission power. We denote the receiving power as $p_i^r$ which is treated as a constant value for all receivers\textsuperscript{3}.

Aiming at achieving high energy efficiency, the energy consumption per information bit for transmitter receiver pair $i$ in each time slot is

$$e_i(p_i, c_i) := \frac{\sum_{k \in L_i} p_i^k + p_i^r}{\sum_{k \in L_i} c_i^k} \quad (5)$$

Let $S_i(p_i, c_i)$ denote the set of all power and rate allocations satisfying QoS requirements and power limit constraints for transmitter $i$, and it is given by

$$S_i(p_i, c_i) = \{ p_i, c_i : p_i \in P, c_i \geq r_i^{tar}, i \in N \} \quad (6)$$

Given the above system assumptions and the objective defined in (5), we end up with the following constrained optimization problem.

$$\min_{p_i^k, c_i^k \in S_i} e_i(p_i, c_i)$$

s.t. $c_i(p_i) \geq r_i^{tar}, \forall i \in N$

$$\sum_{k \in L_i} p_i^k \leq p_i^{max}, \forall i \in N \quad (7)$$

3. Energy efficient resource allocation algorithm

The problem (7) is a combinatorial optimization problem and the objective function is not convex/concave. Constrained optimization techniques can be applied here but with considerable computational complexity. Hence, a two-stage algorithm is proposed in this section to decouple the original problem into an unconstrained problem in order to reduce the search space. After the optimal solution for the unconstrained problem is obtained in stage 1, the power and data rate constraints will be examined in search of the final optimal solution. It should be noted that the solution of the unconstrained problem provides the optimal operating point which can be taken as the benchmark for the system design.

3.1 Unconstrained energy efficient resource allocation

We define the unconstrained energy per bit function as

$$f(\hat{p}_i, \alpha_i) := \frac{\sum_{k \in L_i} \hat{p}_i^k + p_i^r}{\sum_{k \in L_i} \log_2 \left( 1 + \alpha_i^k \cdot \hat{p}_i^k \right)} \quad (8)$$

\textsuperscript{3}In this work, we consider an energy constrained CR ad-hoc wireless network where the throughput requirement is usually not as high as the throughput demanding networks such that the baseband symbol rate is not very high. Thus this baseband power consumption is quite small compared with the power consumption in the RF circuitry. Hence, we neglect the energy consumption of baseband signal processing blocks to simplify the model, and the receiving power equals to the power consumption in the RF circuitry and can be treated as a constant S. Cui (2005)
where $\hat{\cdot}$ is used to represent the variables in the unconstrained optimization domain and $\alpha_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{ik}]$. It is assumed $f(\hat{\cdot}, \alpha_i)$ is a continuous function in $\mathbb{R}^+_M$. We define the unconstrained optimal energy per bit for transmitter $i$ of (8) as $\hat{\zeta}_i = \min f(\hat{\cdot}, \alpha_i)$.

### 3.1.1 Energy efficient waterfilling

**Theorem 1** Given the channel state information $\alpha_i$ and noise power, power allocation $\hat{\rho}_i^* = [\hat{\rho}_1^*, \hat{\rho}_2^*, \ldots, \hat{\rho}_k^*, k \in L_i]$ is defined as the unconstrained optimal power allocation by satisfying

$$f(\hat{\rho}_i^*, \alpha_i) \leq f(\hat{\rho}_i, \alpha_i), \quad \forall \hat{\rho}_i \subset \mathbb{R}^M_+$$

Then the unconstrained optimal power allocation can be obtained by solving the following equations:

$$\hat{\rho}_k^* = \max \left\{ \log_2 e \cdot \hat{\zeta}_i - \frac{1}{\alpha_k}, 0 \right\}$$

$$\hat{\zeta}_i = \frac{\sum_{k \in L_i} \hat{\rho}_k^* + \rho_i}{\sum_{k \in L_i} \log_2 \left( 1 + \alpha_k \cdot \hat{\rho}_k^* \right)}$$

**Proof:** Differentiating $f(\hat{\rho}_i, \alpha_i)$ with respect to $\hat{\rho}_i$ (which stands for the power allocated for transmitter $i$ on subcarrier $k$), we obtain the equations (10). The details of the derivation are given in Appendix A. $\blacksquare$

The value of $\hat{\zeta}_i^*$ can be obtained by using a numerical method which will in turn determine $\hat{\rho}_i^*$. It is observed that $\hat{\rho}_i^*$ has similar type of rate-adaptive / margin-adaptive waterfilling results, and we name it energy-efficient waterfilling. Whereas, the fundamental difference among them lies in the positions of their respective optimal points. The rate-adaptive waterfilling maximizes the achievable data rate under power upper bound, and margin-adaptive waterfilling minimizes the total transmission power subject to a fixed rate constraint W. Yu (2002), both of which achieve their optimality at the boundary of the constraints. On the contrary, the proposed energy-efficient waterfilling selects the most energy-efficient operating point (in other words, it selects the optimal data rate that minimizes the energy consumption per information bit) while adhering to the QoS requirements and power limits. In this case, optimality is usually obtained in the constraint interval rather than on the boundary. In fact, the rate-adaptive and margin-adaptive waterfilling can be considered as special cases of the energy-efficient waterfilling solved. If we set $\sum_{k \in L_i} \rho_k^* = p_{con} \leq p_{i max}^*$ or $\sum_{k \in L_i} \rho_k^* = r_{i tar}$, the energy-efficient allocation problem is reduced to the well explored rate-adaptive or margin-adaptive waterfilling problem.

### 3.1.2 Feasibility region

The existence of the solution for the unconstrained optimization (min $f(\hat{\cdot}, \alpha_i)$) depends on the subcarrier condition $\alpha_k^i$ if we assume other system parameters (e.g. bandwidth, maximal...
Energy Efficient Resource Allocation in Cognitive Radio Wireless Ad Hoc Networks

Fig. 2. Feasible solution vs. subcarrier condition

power, etc.) are fixed. From (10), if we take \( \hat{p}_i \) into the expression of \( \hat{\zeta}^*_i \), we can get

\[
\hat{\zeta}_i = \frac{\Gamma(\hat{p}_i^*) \cdot \log_2(\hat{\zeta}_i) - \sum_{k \in \mathcal{L}_i} \frac{1}{\alpha^k_i} \cdot I(\hat{p}_i^{k*}) + p_i^*}{\Gamma(\hat{p}_i^*) \cdot \log_2(\log_2(\hat{\zeta}_i)) + \sum_{k \in \mathcal{L}_i} \log_2(\alpha^k_i) \cdot I(\hat{p}_i^{k*})}
\]

\[
I(\hat{p}_i^{k*}) = \begin{cases} 
1, & \hat{p}_i^{k*} > 0 \\
0, & \hat{p}_i^{k*} \leq 0
\end{cases}
\]

where \( \Gamma(X) \) is defined as the cardinality of nonzero elements in vector \( X \). The optimal solution \( \hat{\zeta}_i \) can be determined by solving equation (11), and the existence of the optimal solution is influenced by the subcarrier condition \( \alpha^k_i \). This is illustrated in Fig. 2. A unique optimal solution \( (\hat{\zeta}^*_i, 2) \) is obtained when the subcarrier condition is good; while no feasible solution exists when the subcarrier condition is bad. Multiple solutions may be obtained when the subcarrier condition is in the middle range. In this case, only the larger solution \( (\hat{\zeta}^*_i, 3) \) is the feasible solution, and this can be verified by checking the corresponding power allocation, i.e., all the allocated power should be non-negative.

The feasibility condition of the unconstrained optimization problem is given in the following theorem.

**Theorem 2** Denote the maximal optimal solution of \( \hat{\zeta}^*_i \) as \( \hat{\zeta}^{max}_i \) and the channel gain of the best subcarrier as \( \alpha^{\tau}_i \), \( \alpha^{\tau}_i = \max\{\alpha^k_i, \forall k \in \mathcal{L}_i\} \). The feasibility condition for the existence of the optimal solution of the energy efficient waterfilling (10) is given by \( \alpha^{\tau}_i \geq \frac{\ln 2}{\hat{\zeta}^{max}_i} \).

**Proof:** 1) Necessity: From the optimal solution of energy efficient waterfilling (10), it is observed the amount of allocated power is determined by the subcarrier condition \( \alpha^k_i \), specifically, more power should be allocated on better subcarrier. Thus, if the optimal solution exists, at least the power allocated on the best subcarrier should be non-negative, i.e., \( \hat{p}^{\tau}_i = \log_2(\hat{\zeta}^{max}_i) - \frac{1}{\alpha^{\tau}_i} \geq 0 \implies \alpha^{\tau}_i \geq \frac{\ln 2}{\hat{\zeta}^{max}_i} \).
2) Sufficiency: We prove this part by contradiction. If \( \alpha^* \leq \frac{\ln 2}{\hat{\zeta}^{\text{max}}_{i}} \) and still no optimal solution exists, which implies that the power allocated on the entire subcarrier set is negative, i.e., \( \hat{p}^{k*}_{i} < 0 \), then \( \hat{p}^{\text{max}}_{i} - \frac{\ln 2}{\alpha^*} < 0 \implies \alpha^* < \frac{\ln 2}{p^{\text{max}}_{i}} \), which contradicts the condition \( \alpha^* \geq \frac{\ln 2}{\hat{\zeta}^{\text{max}}_{i}} \).

This completes the proof. \( \square \)

Theorem 2 suggests that it is sufficient to check the best available subcarrier in order to determine the feasibility of the unconstrained optimization problem.

3.2 Constrained energy-efficient allocation algorithm

Given the unconstrained optimal solution \( \hat{p}^{*}_{i}, i \in \mathcal{N} \), the previous section offers the optimal operating point with best energy efficiency of each individual user. However, some users may not satisfy their respective data rate and/or power constraints when operating at this point. In this section, we partition the solution space of the constrained optimization problem (7) into four sub-spaces based on the power and data rate constraints, as highlighted in Fig.3.

1. \( \sum_{k \in \mathcal{L}_{i}} \hat{p}^{k*}_{i} \leq p^{\text{max}}_{i} \) and \( \sum_{k \in \mathcal{L}_{i}} c^{k}_{i}(\hat{p}^{k*}_{i}) \geq r^{\text{tar}}_{i} \).

In this case, the unconstrained optimal solution \( \hat{p}^{*}_{i} \) of (10) satisfies the sum-power and rate requirement constraints. Apparently \( \hat{p}^{*}_{i} \) is the optimal solution of the original problem (7).

2. \( \sum_{k \in \mathcal{L}_{i}} \hat{p}^{k*}_{i} \geq p^{\text{max}}_{i} \) and \( \sum_{k \in \mathcal{L}_{i}} c^{k}_{i}(\hat{p}^{k*}_{i}) < r^{\text{tar}}_{i} \) or \( \sum_{k \in \mathcal{L}_{i}} \hat{p}^{k*}_{i} > p^{\text{max}}_{i} \) and \( \sum_{k \in \mathcal{L}_{i}} c^{k}_{i}(\hat{p}^{k*}_{i}) \leq r^{\text{tar}}_{i} \).

In this case, the allocated power has already exceeded the sum-power constraint, but the rate requirement is still not met, even under the optimal subcarrier selection and power allocation. Therefore, there is no feasible solution for the original problem (7).

3. \( \sum_{k \in \mathcal{L}_{i}} \hat{p}^{k*}_{i} < p^{\text{max}}_{i} \) and \( \sum_{k \in \mathcal{L}_{i}} c^{k}_{i}(\hat{p}^{k*}_{i}) < r^{\text{tar}}_{i} \).

If both the power allocated on all subcarriers does not reach the maximal power bound and the data rate requirement is not met, the power should be increased to achieve data rate requirement under the maximal power bound.

Fig. 3. Partition of the solution space of the constrained optimization problem
Based on (7) and (10), we can modify the original problem as

$$\min_{\hat{p}_i^k + \Delta p_i^k \in S_i} \frac{\sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right)}{\sum_{k \in K_i} \log_2 \left( 1 + \alpha_i^k \cdot \left( \hat{p}_i^k + \Delta p_i^k \right) \right)}$$

s.t. \( \sum_{k \in K_i} c_i^k \left( \hat{p}_i^k + \Delta p_i^k \right) \geq r_{tar}^i, \forall i \in \mathcal{N} \)

\( \sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right) \leq p_{max}^i, \forall i \in \mathcal{N} \)

(12)

where \( K_i \) is defined as the selected subcarrier set through the optimal energy efficient waterfilling solution, \( K_i \subset L_i \). If we increase the power on any one of the subcarriers, such as the \( k \)th subcarrier, the corresponding constrained energy consumption per bit can be expressed as

$$\zeta_i^k = \frac{\Delta p_i^k + \sum_{k \in K_i} \hat{p}_i^k + p_i^r}{\sum_{k \in K_i} \log_2 \left( 1 + \alpha_i^k \cdot \hat{p}_i^k \right) + \Delta c_i^k}$$

$$\Delta c_i^k = \log_2 \left( \frac{1 + \alpha_i^k \cdot \hat{p}_i^k + \Delta p_i^k}{1 + \alpha_i^k \cdot \hat{p}_i^k} \right)$$

(13)

From (10), \( \Delta c_i^k \) can be simplified to \( \Delta c_i^k = \log_2 \left( 1 + \frac{\Delta p_i^k}{\log_2 \zeta_i^k} \right) \). It is observed that given the increased power \( \Delta p_i^k \) on subcarrier \( k \), the increased data rate does not rely on its subcarrier condition \( \alpha_i^k \), since \( \log_2 \zeta_i^k \) is a constant value for the entire selected subcarrier set. In other words, for any two subcarrier \( k, l \in K_i \) of transmitter \( i \in \mathcal{N} \), if \( \Delta p_i^k = \Delta p_i^l \), then \( \Delta c_i^k = \Delta c_i^l \). And the constrained energy consumption per bit \( \zeta_i^k \) and \( \zeta_i^l \) will not vary due to different chosen subcarriers. If we presume, in order to reach the data rate requirement \( r_{tar}^i \), the additional required power \( \Delta p_i \) over the selected subcarrier set is known and denoted as \( \Delta p_i = \sum_{k \in K_i} \Delta p_i^k \). Then, problem (12) is equivalent to

$$\min_{\hat{p}_i^k + \Delta p_i^k \in S_i} \frac{\sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right)}{\sum_{k \in K_i} \log_2 \left( 1 + \alpha_i^k \cdot \hat{p}_i^k \right) + \sum_{k \in K_i} \Delta c_i^k}$$

s.t. \( \sum_{k \in K_i} c_i^k \left( \hat{p}_i^k + \Delta p_i^k \right) \geq r_{tar}^i, \forall i \in \mathcal{N} \)

\( \sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right) \leq p_{max}^i, \forall i \in \mathcal{N} \)

(14)

If we assume \( \Delta p_i \) has been pre-determined, in order to minimize energy consumption per bit \( \sum_{k \in K_i} c_i^k \left( \hat{p}_i^k + \Delta p_i^k \right) + \sum_{k \in K_i} \Delta c_i^k \) need to be maximized. In other words, maximizing...
The function \( \sum_{k \in K_i} \Delta c_i^k \) will result in a classical rate-adaptive waterfilling problem.

\[
\max \sum_{k \in K_i} \log_2 \left( 1 + \frac{\Delta p_i^k}{\log_2 \cdot \hat{\tau}_i} \right)
\]

\[
s.t. \sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right) \leq p_i^{\max}, \forall i \in N \tag{15}
\]

Because \( \log_2 \cdot \hat{\tau}_i \) is a constant value for the entire selected subcarrier set \( K_i \), the solution of the above water filling problem implies that the optimal solution \( \Delta p_i^k \) for (15) should be the same for all chosen subcarriers. In other words, given the total required additional power \( \Delta p_i \), the power should be equally allocated on all subcarriers, \( \Delta p_i^k = \frac{\Delta p_i}{\Gamma(K_i)} \). Thus, problem (12) can be rewritten as

\[
\min \sum_{k \in K_i} \left( \hat{p}_i^k + \Delta p_i^k \right)
\]

\[
s.t. \sum_{k \in K_i} \hat{p}_i^k + \Delta p_i \leq p_i^{\max}, \forall i \in N \tag{16}
\]

where \( \sum \Delta c_i^k = \Gamma(K_i) \cdot \log_2 \left( 1 + \frac{\Delta p_i}{\Gamma(K_i) \cdot \log_2 \cdot \hat{\tau}_i} \right) \). Given the unconstrained optimal solution \( \hat{p}_i^k \) from stage 1, (16) can be considered as an objective function in terms of variable \( \Delta p_i \) bounded by \( p_i^{\max} - \sum_{k \in K_i} \hat{p}_i^k \).

**Lemma 1** The constrained energy consumption per bit of problem (16) which is denoted as \( \zeta_i \) is always worse than the unconstrained optimal energy efficiency \( \hat{\zeta}_i \) with respect to the power increase \( \Delta p_i^k \), i.e. \( \zeta_i \geq \hat{\zeta}_i \), \( \forall \Delta p_i \in \mathbb{R}^+ \).

The proof of Lemma 1 is given in Appendix B. Due to the optimality of the unconstrained solution \( \hat{p}_i \), the minimal deviation from \( \hat{\zeta}_i \) will result in the optimal energy efficiency. Thus, the optimal power increase to satisfy the target data rate will be the minimal required additional power as illustrated in Fig.4. Therefore, the optimal required additional power (min \( \Delta p_i \)) to satisfy the data rate requirement \( r_{\text{tar}}^i \) can be calculated as \( \min \Delta p_i = \sum_{k \in K_i} \Delta p_i^k \).

The minimal required additional power \( \Delta p_i^{\min} = \min \Delta p_i \) can be derived by

\[
\log_2 \left( 1 + \frac{\Delta p_i^{\min}}{\Gamma(K_i) \cdot \log_2 \cdot \hat{\tau}_i} \right) = \frac{r_{\text{tar}}^i - \sum_{k \in K_i} \hat{c}_i^k (\hat{p}_i^k)}{\Gamma(K_i)} \tag{17}
\]

From (17), the optimal power increase on \( k \)th subcarrier \( \Delta p_i^k \) is given by

\[
\frac{\Delta p_i^k}{\log_2 \cdot \hat{\tau}_i} = \exp \left( \frac{r_{\text{tar}}^i - \sum_{k \in K_i} \hat{c}_i^k (\hat{p}_i^k)}{\log_2 \cdot \Gamma(K_i)} \right) - 1 \tag{18}
\]
If $\Delta p_{i}^{\text{min}}$ exceeds the remaining power, i.e., $\sum_{k \in K} \hat{p}_{ik} + \Delta p_{i}^{\text{min}} \geq p_{i}^{\text{max}}$, there is no feasible solution for (7). If $\sum_{k \in K} \hat{p}_{ik} + \Delta p_{i}^{\min} \leq p_{i}^{\text{max}}$, the optimal solution for the original problem (7) is

$$p_{i}^{k*} = \hat{p}_{i}^{k*} + \Delta p_{i}^{k*}$$

4. $\sum_{k \in L_{i}} \hat{p}_{ik} > p_{i}^{\text{max}}$ and $\sum_{k \in L_{i}} \zeta_{i}^{k}(\hat{p}_{ik}^{k*}) > r_{i}^{\text{tar}}$.

In this case, the data rate requirement is satisfied but the allocated power exceeds the limit. In order to obtain a feasible solution, the allocated power should be decreased. The derivation of the optimal solution follows similar procedures as given in case 3) and is available in S. Gao (2008).

The inter-relationship and evolvement of the four cases partitioned by the power and data rate constraints are highlighted in Fig.3. Excellent and terrible subcarrier conditions will lead to case 1) (feasible) and case 2) (infeasible), respectively. When the subcarrier conditions are “good”, the solid lines from case 3) and case 4) lead the problem into the feasible region case 1) of the constrained optimization problem when it reaches the maximal power and target data rate bounds, respectively. Whereas, the dashed lines suggest that the problem enters the infeasible region case 2) when the current subcarrier condition cannot accommodate the target data rate under the maximal power limits.

4. Distributed power control

In the previous section, each emerging new user obtains its optimal subcarrier selection and power allocation individually without considering other new users. Although no interference will be introduced to the existing users, due to the non-cooperative behavior of each user, multiple new users may choose the same subcarriers and co-channel interference will be introduced among themselves. In order to maintain user’s QoS, we propose an iterative and distributed algorithm for reaching an equilibrium point among multiple transmitter and receiver pairs based on the distributed power control scheme R. Yates (1995). The distributed
power control algorithm is given by

\[ p^k_i(t + 1) = \min \left\{ \frac{\gamma^k_i}{\gamma^k_i(t)} p^k_i(t), p^\text{max}_i \right\} \]  

(20)

where \( \gamma^k_i \) is the individual target SINR of the \( i^{th} \) transmitter receiver pair over each subcarrier \( k \), which is determined by the constrained optimal solution \( p^*, \gamma^k_i = \exp(\ln2 \cdot c(p^*_i)) - 1 \).

In the power control stage, each node only needs to know its own received SINR \( \gamma^k_i(t) \) at its designated receiver to update its transmission power. This is available by feedback from the receiving node through a control channel. As a result, the proposed scheme is fully distributed. Convergence properties of this type of algorithms were studied by Yates R. Yates (1995). An interference function \( I(P) \) is standard if it satisfies three conditions: positivity, monotonicity and scalability. It is proved by Yates R. Yates (1995) that the standard iterative algorithm \( P(t + 1) = I(P(t)) \) will converge to a unique equilibrium that corresponds to the minimum use of power. The distributed power control scheme (20) is a special case of the standard iterative algorithm.

In summary, the proposed energy efficient spectrum access and resource allocation scheme includes the following steps, as highlighted before in Fig. 1.

**Distributed Energy Efficient Spectrum Access and Resource Allocation**

1. **Initialization**
   - Each transmitter receiver pair obtains their respective available subcarrier set \( L_i \) through spectrum detection.

2. **Individual Energy Efficient Resource Allocation**
   - Each transmitter receiver pair derives its respective unconstrained optimal solution from equation (10).
   - Based on the power limit and data rate constraint, each transmitter receiver pair adjusts its power allocation according to the constrained optimal solution given in Section III. B.
   - Each transmitter receiver pair also calculates its corresponding optimal target SINR \( \gamma^k_i \) based on the constrained optimal solution.

3. **Multiuser Distributed Power Control**
   - Through a control channel, each transmitter acquires the measured SINR \( \gamma^k_i(t) \) from the designated receiver.
   - If \( \gamma^k_i(t) \neq \gamma^k_i \), the transmission power will be updated according to (20).
   - If \( |\gamma^k_i(t) - \gamma^k_i| \leq \epsilon \), \( \forall i \), where \( \epsilon \) is an arbitrary small positive number, the power control algorithm converges to a unique equilibrium point. Otherwise, it is infeasible to accommodate all the new users in the current time slot.

During the power control stage, if the target SINR \( \gamma^k_i \) cannot be maintained when transmitter \( i \) hits its power bound \( p^\text{max}_i \), the network is unable to accommodate all the new users. In this case, a multi-access control (MAC) scheme is required to guarantee the fairness among the users. This will be one of our future efforts.
In this section, we evaluate the performance and convergence of the proposed distributed energy-efficient channel selection and power allocation algorithm. A wireless ad hoc network with cognitive radio capability is considered. Specifically, the parameters of mica2/micaz Berkeley sensor motes G. Anastasi (2004) are employed. The sensor motes operate on 2 AA batteries and the output of each battery is about 1.5 volts, 25000 mAh. The channel gains are assumed to be sampled from a Rayleigh distribution with mean equals to $0.4d^{-3}$, where $d$ is the distance from the transmitter to the receiver. The power bound for the transmission power is 150 mW. The entire spectrum is equally divided into subcarriers with bandwidth 100 kHz. The duration of each timeslot $T_S$ is assumed to be 10 ms in which $L$ bits need to be transmitted. Thus, the target data rate is assumed to be $r_{tar}^i = L/T_S$. The thermal noise power is assumed to be the same over all subcarriers and equals to $10^{-8}$ W.

For each individual user, we first investigate the impact of the target data rate on energy efficiency. We consider a transmitter receiver pair with available subcarrier set $\Gamma(L_i) = 18$, the required data rate $r_{tar}^i = L/T_S$ ranges from $9 \times 10^5$ bps to $1.7 \times 10^6$ bps. In Fig. 5, the square line represents the optimal data rate allocation with the increase of $r_{tar}^i$, while the diamond line represents the required data rate $r_{tar}^i$. It can be observed from Fig. 5 that the optimal rate and power allocation remains approximately unchanged given the channel conditions of the available subcarriers as long as $r_{tar}^i < r_{opt}^i = 1.55 \times 10^6$. After the two lines converge at $L_{opt} = 15500$ bits, the optimal data rate coincides with $r_{tar}^i$, i.e., the required rate can only be obtained at the cost of lower energy efficiency. It is noticeable that $L_{opt}$ is an important system design parameter, and its optimal value can be pre-calculated given the channel conditions.

Fig. 6 illustrates the effect of $L$ (thus the target data rate $r_{tar}^i$ for fixed $T_S$) on energy efficiency. We define $E_i = \zeta_i^* \times L$ as the energy consumption per time slot which is jointly determined by $\zeta_i^*$ and $L$. It is observed that in case 1) with the increase of $L$, $E_i$ increases linearly with respect to $L$ and the energy consumption per bit remains approximately unchanged. When the system

---

4due to numerical round-off errors
Fig. 6. Impact of $L$ to energy efficiency

enters case 3) due to the increase of $r_{tar}^i$, $\xi^*_i$ degrades which suggests that the required data rate $r_{tar}^i$ is satisfied with the expense of energy efficiency.

The impact of the number of available subcarriers on energy efficiency is plotted in Fig. 7. It is shown that the increase of the number of available subcarriers ($\Gamma(L_i)$) improves energy efficiency by providing more available bandwidth. In fact, the total optimal allocated power to satisfy a fixed target data rate is reduced with the increase of $\Gamma(L_i)$. It can be seen in Fig. 7 that the dashed circle line (which represents the unconstrained optimal energy consumption) converges with the constrained energy consumption (solid circle line) when the number of available subcarriers reaches 28. It implies that when the available subcarriers are less than 28, the unconstrained optimal solution corresponds to case 3) in Section 3.2. The system will enter case 1) when $\Gamma(L_i) \geq 28$.

The performance of the proposed energy-efficient waterfilling with respect to network lifetime (which is a critical metric for energy constrained CR ad hoc networks) is investigated. Assuming uniform traffic patterns and persistent traffic flow across the network, we define the network lifetime as $T_l = \frac{E_{max}}{(L \times \xi^*_i)}$, where $E_{max}$ is the maximal energy source of each transmitter. Compared with rate-adaptive and margin-adaptive waterfilling (for transmitting the same amount of information bits in the network), it is observed in Fig. 8 that the proposed scheme outperforms the other two allocation schemes in terms of network lifetime. As the optimal allocated rate approaches the target data rate, energy-efficient waterfilling will converge with margin-adaptive waterfilling as expected. However, since the target data rate in a typical energy constrained ad hoc network is usually low, it is expected that the proposed scheme will improve network lifetime in most applications.

After each new user obtains its optimal subcarrier selection and power allocation independently, distributed power control (20) may be triggered to manage the co-channel interference if multiple new users happen to choose the same subcarriers. The convergence of allocated power is shown in Fig. 9 (including the total required power and the power allocated on two randomly chosen subcarriers of two randomly chosen Tx-Rx pairs). It is observed that the convergence occurs in 3-4 steps.

In this part of the simulation (Fig. 10), the performance of the proposed distributed scheme is
Fig. 7. Impact of number of available subcarriers to energy efficiency

Fig. 8. Performance comparison among different allocation schemes
compared with the centralized optimal solution, where it is assumed that a central controller collects all the $M \times N^2$ channel gain information from all the $N$ new users, and calculates the global optimal solution by considering all the co-channel interference. The case for 8 users and each user with 16 available subcarriers is investigated here S. Gao (2008). It is observed that the proposed distributed scheme (the upper two lines) performs closely to the centralized optimal solution (the middle line). In addition, the competitive optimal solution is also shown in Fig. 10, where each user calculates its own solution without considering co-channel interference (thus optimistic).

6. Conclusion

In this section, a framework of distributed energy efficient resource allocation is proposed for energy constrained OFDMA-based cognitive radio wireless ad hoc networks. A multi-dimensional constrained optimization problem is formulated by minimizing the energy consumption per bit over the entire available subcarrier set for each individual user while satisfying its QoS constraints and power limit. A two-step solution is proposed by first decoupling it into an unconstrained problem, and a constrained partitioning procedure is applied thereafter to obtain the constrained optimal solution by branching the solution space according to power and rate constraints. Co-channel interference may be introduced by concurrent new users and the distributed power control scheme may be triggered to manage the interference and reach the equilibrium point in the multiuser environment.

The proposed spectrum sharing plus resource allocation scheme provide a practical distributed solution for a CR wireless ad hoc network with low computational complexity. It is important to point out that the proposed algorithm for CR networks can be easily modified and applied to multi-channel multi-radio (MC-MR) networks which can be considered as a special case of the CR based wireless networks Y. T. Hou (2007).

In this work, it is assumed that the subcarrier detection is perfect. The effects of detection errors will be investigated in our future work.
Fig. 10. Performance comparison between distributed scheme and global optimality

7. Appendix A

The unstrained optimization problem (8) is

\[
f(\hat{p}_i, \alpha_i) := \sum_{k \in \mathcal{L}_i} \frac{p_i^k + p_i^r}{\log_2 \left( 1 + \alpha_i^k \cdot \hat{p}_i^k \right)}
\]  
(21)

The first order derivative of (21) with respect to \( \hat{p}_i^k \) can be derived as

\[
\frac{\partial f(\hat{p}_i, \alpha_i)}{\partial \hat{p}_i^k} = \frac{1}{\log_2 e} \cdot \left( \frac{\partial \Phi(\hat{p}_i, \alpha_i)}{\partial \hat{p}_i^k} \right)
\]

\[
\Phi(\hat{p}_i, \alpha_i) = \hat{p}_i^k + \sum_{l \in \mathcal{L}_i, l \neq k} \hat{p}_i^l
\]

\[
\ln \left( 1 + \alpha_i^k \hat{p}_i^k + p_i^r \right) + \sum_{l \in \mathcal{L}_i, l \neq k} \ln \left( 1 + \alpha_i^l \hat{p}_i^l \right)
\]  
(22)

If \( k \neq l \), \( c_i(\hat{p}_i^l) \) is taken as constant with respect to \( \hat{p}_i^k \) since the mutual interference between subcarriers is not considered in this work. Therefore, (22) can be expressed as

\[
\frac{\partial \Phi(\hat{p}_i, \alpha_i)}{\partial \hat{p}_i^k} = \sum_{k \in \mathcal{L}_i} \frac{c_i^k(\hat{p}_i^k)}{\ln 2} - \left( \sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p_i^r \right) \frac{\alpha_i^k}{1 + \alpha_i^k \hat{p}_i^k} \left( \sum_{k \in \mathcal{L}_i} \ln \left( 1 + \alpha_i^k \cdot \hat{p}_i^k \right) \right)^2
\]  
(23)
We assume the data rate \( \sum_{k \in L_i} c_i^k (\hat{p}_i^k) \geq 0 \) in this work, thus for \( \frac{\partial f(\hat{p}_i, \alpha)}{\partial \hat{p}_i} = 0 \), (23) can be reduce to

\[
\frac{a_i^k}{1 + a_i^k \cdot \hat{p}_i^k} = \frac{\sum_{k \in L_i} \ln \left( 1 + a_i^k \cdot \hat{p}_i^k + p_i^r \right)}{\sum_{k \in L_i} \hat{p}_i^k} \tag{24}
\]

From (24), we can derive the unconstrained optimal power allocated for transmitter \( i \) over subcarrier \( k \) as

\[
\hat{p}_i^k = \frac{\sum_{k \in L_i} \hat{p}_i^k + p_i^r}{\sum_{k \in L_i} \ln \left( 1 + a_i^k \cdot \hat{p}_i^k \right)} - \frac{1}{a_i^k} \tag{25}
\]

From the definition of unconstrained energy consumption per bit \( \hat{\zeta}_i \), the first term of (25) is in the similar type of \( \hat{\zeta}_i \). If we assume the optimal solution of (A1) does exist (the subcarrier condition resides in the feasible region), there must be a corresponding optimal value of energy per time slot \( \hat{\zeta}_i^* \) with respect to \( \hat{p}_i^* \). Then (25) can be expressed in terms of \( \hat{\zeta}_i^* \) as

\[
\hat{p}_i^* = \log_2 e \cdot \hat{\zeta}_i^* - \frac{1}{a_i^k} \tag{26}
\]

8. Appendix B

The proof of Lemma 1 is given in this appendix. We first define \( f(\Delta p_i) \) as

\[
f(\Delta p_i) = \Delta p_i + \sum_{k \in K_i} \hat{p}_i^k + p_i^r
\]

\[
\sum_{k \in K_i} \Delta c_i^k = \Gamma(K_i) \log_2 \left( 1 + \frac{\Delta p_i}{\Gamma(K_i) \log_2 \hat{\zeta}_i} \right). \quad \text{We denote}
\]

\[
h(\Delta p_i) = \frac{\Delta p_i}{\sum_{k \in K_i} \Delta c_i^k} = \frac{\Delta p_i / \Gamma(K_i)}{\log_2 \left( 1 + \frac{\Delta p_i / \Gamma(K_i)}{\log_2 \hat{\zeta}_i} \right)} \tag{28}
\]

Due to \( x \geq \ln(1+x) \forall x \geq 0 \), we can obtain

\[
\ln 2 \cdot \frac{\Delta p_i / \Gamma(K_i)}{\hat{\zeta}_i^*} \geq \ln \left( 1 + \frac{\Delta p_i / \Gamma(K_i)}{\hat{\zeta}_i^*} \right) \tag{29}
\]

Since \( \Delta p_i, \Gamma(K_i), \) and \( \hat{\zeta}_i^* \geq 0 \) in this work, we get

\[
h(\Delta p_i) = \frac{\Delta p_i / \Gamma(K_i)}{\log_2 \left( 1 + \frac{\Delta p_i / \Gamma(K_i)}{\log_2 \hat{\zeta}_i^*} \right)} \geq \hat{\zeta}_i^* \tag{30}
\]
Based on the definition of $f(\Delta p_i)$, summing $h(\Delta p_i)$ and unconstrained optimal energy per bit ($\hat{\zeta}_i^*$), we have

$$f(\Delta p_i) \geq \hat{\zeta}_i^*, \forall \Delta p_i \in \mathbb{R}^+$$

(31)

Thus, we can conclude the increasing power deteriorates the energy efficiency.

9. Acknowledgment

This research work is supported in part by the U.S. Army Research Office under Cooperative Agreement No. W911NF-04-2-0054. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Office or the U. S. Government.

10. References


