Energy Efficient Spectrum Access in Wireless Cognitive Radio Ad Hoc Networks

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Abstract—In this paper, the resource allocation problem is considered for energy efficient spectrum access in a wireless cognitive radio ad hoc network, where each node is equipped with cognitive radio and has limited battery energy. The network is an OFDMA system operating on time slots. In each slot, the users with new traffic demand will sense the entire spectrum and locate the available subcarrier set. By assuming that a central coordinator has the link gain information of all the new simultaneous transmitting receiving pairs (say through a rendezvous channel), an energy efficient subcarrier selection and power allocation scheme is derived by minimizing energy per bit across the network while considering all the interference from co-channel users, together with QoS requirements and maximal power limit. An $\epsilon$-optimal solution for the unconstrained optimization problem is attained in the first part of the paper by the proposed efficient one-dimensional sub-gradient search algorithm. Then a near-optimal solution for the constrained resource allocation problem is derived by combining the $\epsilon$-optimal solution and distributed power control. The solution provides insights on the properties of energy efficient resource allocation in an ad hoc network which differ from the case of cellular networks, and it can be served as a benchmark for the design of distributed resource allocation schemes for such networks. The results could also be directly applied to networks with certain infrastructure support (such as a cluster based network).

I. INTRODUCTION

Although the U.S. government frequency allocation data [1] shows that there is fierce competition for the use of spectra, especially in the bands from 0 to 3 GHz, it is pointed out in several recent measurement reports that the assigned spectrum are highly under-utilized [2]. Moreover, to access the spectrum in unacquainted environment such as in battlefield applications where the spectrum utilization information may not be directly available to the users [28], fixed spectrum planning will encounter challenges. In order to achieve much better spectrum utilization and viable frequency planning, Cognitive Radios (CRs) are under development to dynamically capture the unoccupied spectrum [3], [4]. Many challenges arise with such dynamic and hierarchical means of accessing the spectrum, especially for the dynamic resource allocation of CR users by adapting their transmission and reception parameters to the varying spectrum condition while adhering to power constraints and diverse quality of service (QoS) requirements (see, for example, [11], [12], [13], [14], [17]).

In this paper, an energy constrained wireless CR ad hoc network is considered, where each node is equipped with cognitive radio and has limited battery energy. One of the critical performance measures of such networks is network lifetime. Hence energy efficient resource management schemes are desired. In this context, the present paper provides a framework for energy efficient resource allocation in wireless CR ad hoc networks that employ orthogonal frequency division multiple access (OFDMA) [7], [8] at the physical layer. OFDMA is well suited for cognitive radio because it is agile in selecting and allocating subcarriers dynamically and facilitates decoding at the receiving end of each subcarrier [24]. In addition, multi-carrier sensing can be exploited to reduce sensing time [5].

The CR OFDMA wireless ad hoc network operates on time slots. Existing users transmit a pilot signal periodically on occupied subcarriers. By detecting the presence of such a pilot signal, emerging CR users can determine the available subcarrier set in a target spectral range, and then select subcarriers with corresponding transmission power based on the proposed algorithm without introducing harmful interference to the existing users [27], [3], [6]. This in turn will improve the energy efficiency of the wireless ad hoc network. It is worth noting that once the new users successfully access the system and start transmission, they will be regarded as primary users and the occupied subcarriers will be excluded from the available subcarrier set.

In this paper, the subcarrier selection and power allocation problem is formulated as a constrained optimization problem. The optimization objective is to minimize the energy consumption per bit across the network (which is defined as the ratio of the total power consumption for transmission and reception throughout the network to its total achieved throughput) while considering the QoS requirements and the (instantaneous) transmission power limits. Compared with the power minimization with respect to target data rate constraints [11] or throughput maximization under power upper bound [12], this objective function, which measures the total energy consumed for reliable information bits transmitted, is particularly suitable for the energy constrained networks where the network lifetime is a critical metric. The energy efficiency objective function considered in this paper has nonquasi-convex/concave nature [23], which implies that standard convex optimization techniques cannot be directly applied here. Hence, we propose a one-dimensional sub-gradient search algorithm to tackle the optimization problem. It is established...
that the unconstrained $\epsilon$-optimal solution can be obtained by
the proposed algorithm with linear computational complexity
in the number of subcarriers (but not of the users).

Resource allocation schemes for multi-carrier systems have
been proposed in many previous studies. In single-user sce-
nario, the optimal allocation strategy with objective to mini-
mize power or maximize throughput is margin-adaptive and
rate-adaptive waterfilling over frequency channels [25], re-
spectively. Whereas, in multi-user case, iterative-waterfilling
(IWF) is presented in [15] as a distributed scheme for re-
source allocation with the objective to enhance the through-
put. In [10], the resource allocation problem is explored
for OFDMA-based wireless network by directly adopting
distributed power control scheme for the power and bits
allocation on all subcarriers to improve power efficiency. In
the context of CR enabled wireless sensor network [11], a two-
step algorithm is proposed to tackle the allocation problem:
channel assignment with objective of minimizing transmission
power and channel contention to reserve the subcarrier set for
transmission by intended transmitters, while the interference
spectrum mask is assumed to be known a priori. In multi-
carrier direct-sequence CDMA (DS-CDMA) cellular system,
a non-cooperative power control game for resource allocation
with respect to maximizing the energy efficiency is proposed
in [23] which leads to the best subcarrier selection scheme
for each user respectively. Considering the scenario in ad
hoc networks where there exist multiple transmitting receiving
pairs, we show in this work that the optimal energy efficient
resource allocation scheme may require multiple subcarriers
to be chosen simultaneously by one user-pair and certain
subcarriers may be shared among multiple user-pairs.

In our recent work [27], a distributed energy efficient
spectrum access and resource allocation algorithm is proposed
in the infrastructureless environment. For the sake of provid-
ing a benchmark to evaluate the performance of distributed
strategies, a centralized energy efficient spectrum access and
resource allocation model is considered here and a near-
optimal solution is provided in this paper by assuming a
central agent has the knowledge of the underlying channel
fading distributions of all the new simultaneous transmitting
receiving pairs (say through a rendezvous channel). The global
(near) optimal solution obtained in this work provides valuable
insights of the properties of the optimal energy efficient
resource allocation in a multi-user-pair multi-carrier wireless
ad hoc network. Furthermore, when infrastructure support is
available (such as in a cluster based network) and a dedicated
common control channel can provide the central agent (say,
the cluster head) with the required channel conditions, the
centralized scheme can be implemented as well.

The remainder of this paper is organized as follows. In
section II, the system model of energy efficient resource
allocation problem in CR network is presented. An optimal
allocation algorithm without considering the power and QoS
constraints is proposed firstly in section III, then a distributed
power control based algorithm is designed to satisfy the con-
straints if they are not met. Section IV contains a case study
of two user with two subcarriers to demonstrate the superior
performance of the proposed algorithm. Further simulation
results and analysis are provided in section V. Section VI gives
concluding remarks.

II. SYSTEM MODEL

We consider an energy constrained CR OFDMA network of
$N$ communicating user-pairs. Both transmitter $i$ and receiver
$j$ is indexed by $N_i := \{1, 2, ..., N\}$. If $j = i$, receiver $j$ is said
to be the intended receiver of transmitter $i$. The transmission
system is assumed to be a time-slotted OFDMA system with
fixed time slot duration $T_s$. Slot synchronization is assumed to
be achieved through a beaconing mechanism. Before each time
slot, a guard interval is inserted to achieve synchronization,
perform spectrum detection as well as resource allocation
(based on the proposed scheme). Inter-carrier interference
(ICI) caused by frequency offset of the side lobes is not
considered in this work (which can be mitigated by windowing
the OFDM signal in the time domain or adaptively deactivating
adjacent subcarriers [18]).

A frequency selective Rayleigh fading channel is assumed
at the physical layer, and the entire spectrum is appropri-
ately divided into $M$ subcarriers to guarantee each sub-
carrier experiencing flat Rayleigh fading [9]. We label the
subcarrier set available to the transmitter receiver pair $i$
after spectrum detection by $L_i \subset \{1, 2, ..., M\}$. Let $G := \{G_{i,j}, i,j \in N, k \in L_i\}$ denote the subcarrier fading coeffi-
cient matrix, where $G_{i,j}$ stands for the sub-channel coeffi-
cient gain from transmitter $i$ to receiver $j$ over subcarrier
$k$. $G_{i,j,k} = |H_{i,j,k}(f)|^2$, where $|H_{i,j,k}(f)|$ is the transfer func-
tion [26]. It is assumed that $G$ is available to a central
agent and $G$ adheres to a block fading channel model which
remains invariant over blocks (coherence time slots) of size
$T_S$ and uncorrelated across successive blocks. The noise is
assumed to be additive white Gaussian noise (AWGN), with
variance $\sigma^2_{n,k}$ over subcarrier $k$ of receiver $i$. We define
$P := \{p^i_k, p^k \geq 0, i \in N, k \in L_i\}$ as the transmission power
allocation matrix for all users in $N$ over the entire available
subcarrier set $\bigcup_{i \in N} L_i$, where $p^i_k$ is the power allocated over
subcarrier $k$ for transmitter $i$. For each transmitter $i$, the power
vector can be formed as

$$p_i := [p^1_i, p^2_i, ..., p^N_i]^T.$$  

If the $k$th subcarrier is not available for transmitter $i$, $p^i_k = 0$.
Each node is not only energy limited but also has peak power
constraint, i.e., $\sum_{k \in L_i} p^i_k \leq p^{i,max}_k$. The set of all feasible
power vector of transmitter $i$ is denoted by $P_i$

$$P_i := \left\{ p_i \in \prod_{k \in L_i} [0, p^{i,max}_k], \sum_{k \in L_i} p^i_k \leq p^{i,max}_k, p^i_k \geq 0 \right\}$$  

The signal to interference plus noise ratio (SINR) of receiver
$i$ over subcarrier $k$, $\gamma_i^k$, can be expressed as

$$\gamma_i^k(p^k_i) = \alpha_i^k(p^k_i) \cdot p^k_i$$
$$\alpha_i^k(p^k_i) = \frac{G_{i,i,k}^k}{\sum_{j \neq i, j \in N} G_{j,i,k}^2 + \sigma^2_{n,k}}$$
where $\alpha^k_i$ is defined as the channel state information (CSI) which treats all interference as background noise. $\alpha^k_i$ can be measured at the receiver side and is assumed to be known by the corresponding transmitter through a reciprocal common control channel.

When all users divide the spectrum in the same fashion without coordination, it is referred to as a Parallel Gaussian Interference Channel [19] which leads to a tractable inner bound to the capacity region of the interference model. The achievable maximum data rate for each user may be calculated using Shannon’s capacity formula

$$\frac{c_i(p_i)}{B_i} = \sum_{k \in \mathcal{L}_i, p_i^k \in \mathcal{P}_i} \log_2 \left( 1 + \frac{G^k_i \cdot p_i^k}{\sum_{j \neq i, j \in N} G^k_{j,i} \cdot p_j^k + \sigma^2_{i,k}} \right) \quad (4)$$

where $B_i^k$ is the equally divided subcarrier bandwidth for transmitter $i$. Without loss of generality, $B_i^k$ is assumed to be unity in this work. The noise is assumed to be independent of the symbols and has variance $\sigma^2$ for all receivers over entire available subcarrier set. Furthermore, all communicating transmitter and receiver pairs are assumed to have diverse QoS requirements specified by $\sum_{k \in \mathcal{L}_i, p_i^k \in \mathcal{P}_i} c_i^k \geq r^\text{tar}_i$, where $r^\text{tar}_i$ is the target data rate of transmitter $i$.

In an energy constrained network (such as a wireless sensor network), reception power is not negligible since it is generally comparable to the transmission power [21], [20], [22]. In this work, we denote the receiving power as $p^r_i$ which is treated as a constant value for all receivers [20].

Aiming at achieving high energy efficiency, the energy consumption per information bit for transmitter receiver pair $i$ in each time slot is

$$e_i(p_i, c_i) = \frac{\sum_{k \in \mathcal{L}_i, p_i^k \in \mathcal{P}_i} c_i^k \cdot p_i^k + p^r_i}{\sum_{k \in \mathcal{L}_i, c_i^k}} \quad (5)$$

Let $\mathcal{S}_i(p_i, c_i)$ denote the set of all power and rate allocations satisfying QoS requirements and power limit constraints for transmitter $i$, and it is given by

$$\mathcal{S}_i(p_i, c_i) = \{ p_i, c_i : p_i \in \mathcal{P}_i, c_i \geq r^\text{tar}_i, i \in \mathcal{N} \} \quad (6)$$

Given the above system assumptions and the objective defined in (5), we end up with the following constrained optimization problem.

$$\min_{\mathcal{S}_i(p_i, c_i), p_i^k \in \mathcal{P}_i} \left[ \sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} \log_2 \left( 1 + \frac{G^k_i \cdot p_i^k}{\sum_{j \neq i, j \in N} G^k_{j,i} \cdot p_j^k + \sigma^2_{i,k}} \right) \right) \right]$$

subject to

$$\sum_{k \in \mathcal{L}_i} \log_2 \left( 1 + \frac{G^k_i \cdot p_i^k}{\sum_{j \neq i, j \in N} G^k_{j,i} \cdot p_j^k + \sigma^2_{i,k}} \right) \geq r^\text{tar}_i, \forall i \neq j \in \mathcal{N}$$

$$\sum_{k \in \mathcal{L}_i} p_i^k \leq p_i^\text{max}, \forall i \in \mathcal{N}$$

$$\sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} c_i^k \right)$$

III. CENTRALIZED ALLOCATION SCHEME

In order to locate the optimal operating point which can be taken as the benchmark, the unconstrained optimization problem is investigated firstly. A one-dimensional search algorithm is presented to derive the unconstrained $\epsilon$-optimal solution. Then distributed power control based algorithm is proposed to achieve the power and data rate constraints.

A. Unconstrained Energy Efficient Allocation Algorithm

We define the unconstrained energy per bit function as

$$f(p_i, \alpha_i) := \frac{\sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} c_i^k \cdot p_i^k + p^r_i \right)}{\sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} G^k_i \cdot p_i^k + \sigma^2_{i,k} \right)}$$

where $\alpha$ is used to represent the variables in the unconstrained optimization domain. Due to the nonnegativity of the power assumption and realized data rate, it can be assumed that $f(p_i, \alpha_i)$ is continuous and differentiable on $\mathbb{R}^M \times \mathbb{R}^N$. Given the channel gain matrix $G$ and noise power, power allocation $\hat{p}^e = \{ \hat{p}_1, \hat{p}_2, \ldots, \hat{p}_M \}$ is defined as the optimal point by satisfying

$$f(p_i^*, \alpha_i^*) \leq f(p_i, \alpha_i), \forall p_i \in \mathbb{R}^M \quad (8)$$

Denote the optimal energy per bit as $\hat{\zeta} = f(p_i^*, \alpha_i^*)$ and differentiating $f(p_i, \alpha_i)$ with respect to $p_i^k$, we obtain

$$\frac{\partial f(p_i, \alpha_i)}{\partial p_i^k} = \frac{\sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} c_i^k \cdot p_i^k \right) - \sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p^r_i \right) \left( \frac{\partial \Phi(p_i^k)}{\partial p_i^k} \right)}{\log_2 \hat{p}_i^k} \quad (9)$$

where $\Phi(p_i^k) := \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{L}_i} c_i^k \cdot (\hat{p}_i^k)$ is the achievable data rate throughout the entire network. The derivation of (9) is given in Appendix A. It is observed that equation (9) can be re-organized in terms of $\hat{\zeta}$

$$\ln 2 \cdot \hat{\zeta} = \frac{\hat{\alpha}_i^k (\hat{p}_i^k)}{1 + \hat{\alpha}_i^k (\hat{p}_i^k) \cdot \hat{p}_i^k} + \sum_{v \in \mathcal{N}, v \neq i} \left( \frac{\partial \hat{\alpha}_v (\hat{p}_j^k)}{\partial p_v^k} \hat{p}_v^k \right)$$

$$\hat{\zeta} := \frac{\sum_{i \in \mathcal{N}} \left( \sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p^r_i \right)}{\sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{L}_i} c_i^k (\hat{p}_i^k)} \quad (10)$$

Now instead of relying on $(M \times N)$ dimensional search for the optimal solution $\hat{p}^e$, (10) can be grouped in terms of...
subcarrier if $\hat{\zeta}^*$ is considered to be a constant (and can be obtained by one-dimensional search). Accordingly, the original $(M \times N)$ dimensional optimization problem is reduced to $M$ sets of equations with $N$ polynomial equations in each set, and each set of equations can be solved individually. Note that due to the nonconvexity of $f(p_i, \alpha_i)$, the solution obtained by the differentiation may be only locally optimal. Whereas the global minimizer can be found by examining the nonnegativity of the allocated power and the corresponding energy efficiency $\hat{\zeta}$ during the search process.

In this work, we propose an efficient one-dimensional search algorithm (sub-gradient iterations) for calculating $\hat{\zeta}^*$ instead of $\hat{p}_i^*$, since $\hat{p}_i^*$ can be directly determined by $\hat{\zeta}^*$. Thus, the computational complexity of the original problem is greatly reduced. Specifically, we adopt the following sub-gradient projection iterations (indexed by n) for locating $\hat{\zeta}^*$

$$\hat{\zeta}^*(n + 1) = \hat{\zeta}^*(n) + \beta(n) \left( \sum_{i \in C} \sum_{k \in C_i} g_k^i(\hat{\zeta}^*(n)) \sum_{i \in C} \sum_{k \in C_i} c_k^i(\hat{\zeta}^*(n)) - \hat{\zeta}^*(n) \right)$$

where $\beta(n)$ is a positive step size, $g_k^i(\hat{\zeta}^*(n))$ is defined as the function of $\hat{\zeta}^*(n)$ for determining $\hat{p}_k^i(n)$, i.e., $\hat{p}_k^i = g_k^i(\hat{\zeta})$. $c_k^i(n)$ denotes the achievable data rate by $\hat{\zeta}^*(n)$.

In order to implement the algorithm efficiently, the search range for $\hat{\zeta}$ should be narrowed down to reduce the number of iterations. We define $\hat{\zeta}_{\text{int}}$ as the initial point for searching $\hat{\zeta}^*$. In the proposed algorithm, we choose $\hat{\zeta}_{\text{int}}$ as the competitive optimal energy per bit (which corresponds to the case that there is no multi-access interference among transmitter receiver pairs). The derivation of $\hat{\zeta}_{\text{int}}$ can be found in [27]. Simulations demonstrate the above search algorithm with proper initialization converges very fast (mostly in 3-4 steps).

**Unconstrained Resource Allocation Algorithm**

1) **Initialization**
   - Initialize the unconstrained energy per bit $\hat{\zeta}(n)=0$.
   - Initialize temporary energy per bit $\zeta_{\text{int}}=0$.
   - Initialize temporary transmission power $\hat{p}_{\text{temp}}=\phi$.

2) **Sub-Gradient search algorithm of $\hat{\zeta}^*$**
   - Set $\hat{\zeta}(n)$ to the initial value $\hat{\zeta}_{\text{int}}$.
   - Determine the tentative power allocation $\hat{p}_{\text{temp}}$ by $\hat{\zeta}(n)$.
   - Derive the tentative energy efficiency $\zeta_{\text{int}}$ by $\hat{p}_{\text{temp}}$.
   - Examine whether $\zeta_{\text{int}} - \hat{\zeta}(n) \leq \epsilon$, $\epsilon$ is an arbitrary small real number.
   - If yes, stop and the unconstrained optimal energy per bit $\hat{\zeta}^* = \hat{\zeta}_{\text{int}}$.
   - If no, update $\hat{\zeta}(n)$ to $\hat{\zeta}(n + 1)$ according to sub-gradient projection iterations and go to the beginning of step 2).

**B. Constrained Energy Efficient Allocation Solution**

With the unconstrained $\epsilon$-optimal solution $\hat{\zeta}^*$ which can determine the unconstrained optimal power $\hat{p}$ and data rate $c(\hat{p})$, the previous section offers the optimal operating point of the system. This optimal solution can be regarded as the benchmark for the system design with best energy efficiency. However, some users may not satisfy their respective data rate and/or power constraints when operating at this optimal point. In this section, the emerging transmitter receiver pairs are categorized into four subsets:

- $\mathcal{N}_A \subset \mathcal{N}$, if $\sum_{k \in C_i} p_k^i \leq p_i^{\text{max}}$, $\sum_{k \in C_i} \Delta p_k^i \geq r_i^\text{tar}$
- $\mathcal{N}_B \subset \mathcal{N}$, if $\sum_{k \in C_i} p_k^i \geq p_i^{\text{max}}$, $\sum_{k \in C_i} \Delta p_k^i \leq r_i^\text{tar}$
- $\mathcal{N}_C \subset \mathcal{N}$, if $\sum_{k \in C_i} p_k^i \leq p_i^{\text{max}}$, $\sum_{k \in C_i} \Delta p_k^i \leq r_i^\text{tar}$
- $\mathcal{N}_D \subset \mathcal{N}$, if $\sum_{k \in C_i} p_k^i \geq p_i^{\text{max}}$, $\sum_{k \in C_i} \Delta p_k^i \geq r_i^\text{tar}$

After each transmitter obtains its respective unconstrained optimal point $(\hat{p}^*, \hat{c}^*)$, and if there exist any user not in $\mathcal{N}_A$, the constrained resource allocation algorithm will be provoked for each transmitter by performing distributed power control [16]

$$p_i^k(t + 1) = \min \left( \frac{a_i^k(\hat{p}_i^k(t)) \cdot \hat{p}_i^k(t)}{\gamma_i}, p_i^{\text{max}} \right)$$

where the individual target SINR of each transmitter over each selected subcarrier $\gamma_i^{k*}$ need to be determined.

1) $i \in \mathcal{N}_A$
   - Since transmitter $i \in \mathcal{N}_A$ satisfies both the power and rate constraints, according to (3) the target SINR can be expressed as
     $$\gamma_i^{k*} = \alpha_i^k(\hat{p}_i^k) \cdot \hat{p}_i^k, \forall i \in \mathcal{N}_A$$

2) $i \in \mathcal{N}_B$
   - In this case, there is no feasible solution for transmitter $i \in \mathcal{N}_B$. Thus, transmitters in $\mathcal{N}_B$ should suspend the transmission in the current time slot.

3) $i \in \mathcal{N}_C$
   - For each transmitter $i$ that does not satisfy the rate requirement, the transmission power needs to be increased to reach $r_i^\text{tar}$. In this paper, we provide an near-optimal solution for SINR derivation with tight performance to the optimal one. Owing to the optimality of the unconstrained power allocation $\hat{p}$, it can be inferred that larger offset from $\hat{p}$ will result in further degradation in energy efficiency. Accordingly, the power increase of each user should cease when they meet their QoS requirements. However, if the power bound has been reached before the data rate requirement is met, this user should suspend the transmission in current slot.

   The increased data rate for transmitter $i \in \mathcal{N}_C$ can be expressed as $r_i^\text{tar} = r_i^\text{tar} - \sum_{k \in C_i} \Delta p_k^i$. If we first investigate the additional power increase problem for each user $i \in \mathcal{N}_C$ without considering co-channel interference, the data rate increase can be calculated as

$$\Delta c_i^k = \log_2 \left( \frac{1 + \alpha_i^k(\hat{p}_i^k) \cdot \hat{p}_i^k + \Delta p_k^i}{1 + \alpha_i^k(\hat{p}_i^k) \cdot \hat{p}_i^k} \right)$$
To minimize the energy efficiency degradation due to the deviation from the optimal point $\hat{p}$, the increased power $\Delta p_i$ for user $i \in \mathcal{N}_C$ should be minimized which leads to the following problem

$$
\min \sum_{k \in \mathcal{L}_i} \Delta p_i^k, \forall i \in \mathcal{N}_C
$$

$$
s.t. \sum_{k \in \mathcal{L}_i} \Delta c_i^k = \Delta r_i^{tar}, \forall i \in \mathcal{N}_C
$$

(14)

The optimal solution of (14) can be obtained as

$$
\Delta c_i^k = \log_2 \left( \beta_i^k \cdot \exp \left( \frac{\ln 2}{M_i} (\Delta r_i^{tar} - \sum_{k \in \mathcal{L}_i} \log_2 \beta_i^k) \right) \right)
$$

$$
\beta_i^k = \frac{\log_2 \beta_i^k (p_i^{k*}) - \hat{p}_i^k}{1 + \hat{p}_i^k (p_i^{k*}), \forall i \neq j \in \mathcal{N}_C},
$$

Define the number of non-zero components in a vector $x$ as $l(x)$, then $l(\hat{p}_i^k) = M_i$. The derivation for $\Delta c_i^k$ is given in Appendix B. The target SINR on each subcarrier is given by

$$
\gamma_i^k = \exp \left( \Delta c_i^k + c_i^k (p_i^{k*}) \right) - 1
$$

(15)

4) $i \in \mathcal{N}_D$

Since the decrease of power benefits the peers by lowering the co-channel interference, each user $i \in \mathcal{N}_D$ only needs to decrease the power individually to reach the power bound. When the power limits are met, the user should stop which will lead to the minimal offset from $\hat{p}_i^k$. If the user cannot satisfy the data rate constraint at the power limit, it should suspend in the current time slot.

If we investigate the power decrease of each transmitter $i \in \mathcal{N}_D$ without considering the co-channel interference, the data rate decrease can be calculated as

$$
\Delta c_i^k = \log_2 \left( \frac{1 + \hat{p}_i^k (p_i^{k*}) \cdot \hat{p}_i^k}{1 + \hat{p}_i^k (p_i^{k*}) (p_i^k - \hat{p}_i^k)} \right)
$$

(16)

Define $\Delta r_i^{tar} = p_i^{max} - \sum_{k \in \mathcal{L}_i} \hat{p}_i^k$. In order to minimize the energy efficiency degradation, the decreased data rate $(\Delta c_i^k)$ should be minimized

$$
\min \sum_{k \in \mathcal{L}_i} \Delta p_i^k, \forall i \in \mathcal{N}_D
$$

$$
s.t. \sum_{k \in \mathcal{L}_i} \Delta p_i^k = \Delta r_i^{tar}, \forall i \in \mathcal{N}_D
$$

(17)

The optimal solution of (17) is

$$
\Delta r_i^{k*} = \left[ \hat{p}_i^k + \frac{1}{\hat{p}_i^k (p_i^{k*})} \cdot \frac{\Lambda}{M_i} \right]^{+}
$$

$$
\Lambda = \left( 2 \cdot \sum_{k \in \mathcal{L}_i} \hat{p}_i^k + \sum_{k \in \mathcal{L}_i} \frac{1}{\hat{p}_i^k (p_i^{k*})} \cdot p_i^{max} \right)
$$

(18)

where $l(\hat{p}_i^k) = M_i$. The derivation is given in Appendix C. The target SINR is

$$
\gamma_i^{k*} = \hat{p}_i^k (p_i^{k*}) \cdot \frac{\Lambda}{M_i} - 1
$$

(19)

To achieve the power and rate constraints, each transmitter in the network conducts power control scheme (11) according to their respective target SINR. It is noticeable that co-channel interference is ignored in case 3) and 4) when calculating the target SINR. However, the co-channel interference will be taken into account automatically when all the users participate in the distributed power control (11). Therefore, a near optimal solution is provided in this section that satisfying the QoS and power constraints.

IV. CASE STUDY

In order to evaluate the proposed algorithm and provides insights of the properties of the optimal solution, we concentrate on a system with two users and two subcarriers in this section $(N = M = 2)$. The optimal power allocation vector is $p_i^* = [p_1^*, p_2^*, p_1^{*}, p_2^{*}]$. The power and rate constraints are $p_{1,\max} = p_{2,\max} = 50$mW, $r_{1,\max} = r_{2,\max} = 0.4$Mbps. Following (10), the optimal power allocation for user 1 on subcarrier 1 is

$$
\frac{\ln 2}{1 + \hat{p}_i^1 \cdot p_i^{1*}} = \frac{\hat{p}_i^1 \cdot p_i^{1*} \cdot G_{i1}^2}{G_{i1}^2 \cdot \hat{p}_i^1 + \sigma^2}, \quad \hat{\alpha}_i^1 = \frac{G_{i1}^2}{G_{i1}^2 \cdot \hat{p}_i^1 + \sigma^2}
$$

(20)

Similar expressions can be obtained for other users and subcarriers. As stated before, the power allocation can be grouped in terms of subcarrier. The channel gain matrices for subcarrier 1 and 2 are

$$
G^1 = \begin{pmatrix} G_{11}^1 & G_{12}^1 \\ G_{21}^1 & G_{22}^1 \end{pmatrix}, \quad G^2 = \begin{pmatrix} G_{11}^2 & G_{12}^2 \\ G_{21}^2 & G_{22}^2 \end{pmatrix}
$$

(21)

The numerical values are

$$
G^1 = \begin{pmatrix} 6.38 \times 10^{-6} & 2.1 \times 10^{-7} \\ \frac{1.18 \times 10^{-6}}{4.13 \times 10^{-6}} & \frac{8.23 \times 10^{-6}}{9.27 \times 10^{-6}} \end{pmatrix}, \quad G^2 = \begin{pmatrix} 3.19 \times 10^{-7} & 1.53 \times 10^{-7} \\ \frac{2.73 \times 10^{-7}}{9.27 \times 10^{-6}} & \frac{2.73 \times 10^{-7}}{9.27 \times 10^{-6}} \end{pmatrix}
$$

We assume equal noise over all subcarriers and $\sigma^2 = 10^{-8}$. Following the proposed algorithm, the optimal power allocation is $P^* = [13.7, 11.5, 13.5, 14]$ (in mW) and the optimal energy per bit $\zeta^* = 1.356 \times 10^{-7}$ joule/bit as illustrated in Fig.1.
Remarks: Given the subcarriers conditions and the power and rate constraints, it should be noted that the best energy efficiency is achieved when the two subcarriers are shared by the two users. It demonstrates that the best energy efficiency is usually achieved when the subcarriers are accessed by multiple transmitters simultaneously even though multi-access interference may be introduced. Indeed, the “speed” of the increase of data rate exceeds that of the power before the system reach the optimal point, which will lead to better performance. This is because that the contribution to the data rate from more bandwidth is much greater compared to the degradation induced by the co-channel interference. On the other hand, more allocated power beyond the optimal point will degrade the energy efficiency since the introduced interference will now become dominant.

In [23], the energy efficient power allocation scheme in multicarrier CDMA cellular system is explored, which suggests that energy efficiency is maximized when each user transmits only over its “best” subcarrier. The problem can be stated as

\[
\begin{align*}
\min_{k} & \quad \frac{p_{k1}^* + p_r}{\log_2 (1 + \alpha_{k1}^* \cdot p_{k1}^* )} \\
\text{s.t.} & \quad p_{k1}^* \leq \frac{\max \{ p_{k} \} }{\log_2 (1 + \alpha_{k1}^* \cdot p_{k1}^* )} \\
& \quad \log_2 (1 + \alpha_{k1}^* \cdot p_{k1}^* ) \geq r_{tar}, \ k \in L_i, \forall i \in N
\end{align*}
\]

From the subcarrier conditions shown in Fig.2, it is apparent that user 1 should choose subcarrier 1 and user 2 should select subcarrier 2. Therefore, the optimal power allocation for best subcarrier selection is

\[
\begin{align*}
p_{k1}^* &= \zeta^* - \frac{1}{\alpha_{k1}^*} \\
\zeta^* &= \frac{p_{k1}^* + p_r}{\log_2 (1 + \alpha_{k1}^* \cdot p_{k1}^* )}
\end{align*}
\]

From (23), \( \zeta_1^* = 1.8 \times 10^{-7} \), \( \zeta_2^* = 1.6 \times 10^{-7} \) as shown in Fig.2. Unlike cellular systems employing multi-carrier modulation, where it has been shown that best subcarrier selection is optimal [23], multiple subcarriers may need to be chosen simultaneously in ad hoc networks to achieve best energy efficiency. Even though co-channel interference is introduced, the overall energy efficiency is still superior to the best subcarrier selection approach.

V. Simulation Result

A. Simulation Setup

In this section, the performance of the proposed scheme is evaluated with respect to the number of available subcarriers and the convergence of power control is demonstrated. We consider a wireless ad hoc network composed of sensor nodes, and the experimental data of mica2/micaz Berkeley motes is adopted [22]. We assume the sensor motes operate on 2 AA batteries and the output of each battery is about 1.5 volts, 25000 mAh. The channel gains are assumed to be sampled from a Rayleigh distribution with mean equals to \( 0.4d^{-3} \), where \( d \) is the distance from the transmitter to the receiver. The receiving power of each receiver is assumed to be constant at 48 mW and the power bound for the transmission power is 50 mW. The entire spectrum is equally divided into subcarriers with bandwidth 100 KHz. The duration of each time slot \( T_S \) is assumed to be 10ms in which \( L \) bits need to be transmitted. Thus, the target data rate is \( r_{tar} = L/T_S \). The thermal noise power is assumed to be the same over all subcarriers and equal to \( 10^{-8} \)W.

B. Performance Evaluation of the Unconstrained Algorithm

We consider the system with two users who have the same data rate requirement \( 0.2M/s \), i.e., \( L = 2K \) bits. The unconstrained optimal results can be regarded as the benchmark for an energy-constrained system design as it provides the most energy efficient operating point of the system. The influence of the number of subcarriers on energy efficiency when applying the proposed unconstrained algorithm is given in Fig.3. It is observed that with the increase of the number of subcarriers, energy efficiency is improved from \( 7.86 \times 10^{-8} \) to \( 2.2 \times 10^{-7} \) at the expense of more available bandwidth.

In Fig.4, the optimal unconstrained allocated power ranges from 21.2mW to 37.1mW for transmitter 1 (Tx1) and 20.5mW to 34.3mW for transmitter 2 (Tx2) over all subcarriers. However, the average allocated power decreases from 21.2mW to 7.7mW for transmitter 1 and 20.5mW to 6.9mW for transmitter 2. It is observed that with the increase of the number of subcarriers, the total allocated power and the total achieved data rate will increase as well. On the other hand, the average allocated power per subcarrier decreases “faster” than the reduction of the average data rate per subcarrier. This is because the interference condition across the network improves as more bandwidth becomes available. The users may take advantage of less noisy channels and spread power into them to improve energy efficiency.

C. Performance Evaluation of Constrained Algorithm

We now evaluate the performance of the constrained resource allocation algorithm in case that the power or data rate constraints are not met. We use the same channel gain distribution and system parameters as given in section IV. It is assumed there are two users in the system with data rate requirement of 0.55Mbps and 0.5Mbps, respectively. After executing the unconstrained algorithm, user 1 does not satisfy the data rate requirement while user 2 does, i.e., user 1 \( \in \mathcal{N}_C \)
In this work, a framework of energy efficient resource allocation is proposed for energy constrained OFDMA-based cognitive radio wireless ad hoc networks. A multi-user multi-subcarrier non-convex constrained optimization problem is formulated by minimizing the energy consumption per bit across the network while considering individual user’s QoS constraints and power limit. The optimal solution obtained from the unconstrained algorithm provides insights on the properties of optimal energy efficient resource allocation in an ad hoc network which differ from the case of cellular networks.

The centralized solution presented in this work may be served as a benchmark for the design of distributed resource allocation schemes for such networks [27]. Note that when the network is unable to accommodate all the users in the current time slot, a multi-access control is needed to maintain fairness among users. However, this is out of the scope of this paper. The proposed two-step algorithm for the constrained optimization problem only provides a sub-optimal solution. The optimal solution with reasonable computational complexity will be investigated in our future work. In addition, large scale simulation experiments will be conducted as well.

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VII. APPENDIX A

Based on the unconstrained energy per bit function, 
\[
    f(\hat{p}, \hat{\alpha}) := \frac{\sum_{i \in N} \left( \sum_{k \in L_i} \hat{p}_i^k + p_i^* \right)}{\sum_{i \in N, j \neq i} \log_2 \left( 1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k \right)}
\] 
(24)

the first order derivative of (24) is given by 
\[
    \frac{\partial f(\hat{p}, \hat{\alpha})}{\partial \hat{p}_i^k} = \sum_{i \in N} \sum_{k \in L_i} c_i^k (\hat{p}_i^k) - \sum_{i \in N} \left( \sum_{k \in L_i} \hat{p}_i^k + p_i^* \right) \left( \frac{\partial \Phi(\hat{p}_i^k)}{\partial \hat{p}_i^k} \right) \left( \sum_{i \in N, j \neq i} \log_2 \left( 1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k \right) \right)
\] 
(25)

From (4), the data rate for transmitter \( i \) over subcarrier \( k \) is defined as
\[
    c_i^k (\hat{p}_i^k) = \log_2 \left( 1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k \right), \quad \forall i \neq j \in N, \forall k \in L_i
\]
The throughput of the entire network is defined as \( \Phi(\hat{p}_i^k) \), which is assumed to be greater than zero in this work
\[
    \Phi(\hat{p}_i^k) = \sum_{i \in N} \sum_{k \in L_i} c_i^k (\hat{p}_i^k), \quad \forall i \in N, \forall k \in L_i
\] 
(26)

Thus (25) can be simplified to
\[
    \frac{\partial f(\hat{p}, \hat{\alpha})}{\partial \hat{p}_i^k} = \Phi(\hat{p}_i^k) - \sum_{i \in N} \left( \sum_{k \in L_i} \hat{p}_i^k + p_i^* \right) \left( \frac{\partial \Phi(\hat{p}_i^k)}{\partial \hat{p}_i^k} \right)
\] 
(27)

Differentiating \( \Phi(\hat{p}_i^k) \) with respect to \( \hat{p}_i^k \) will lead to
\[
    \frac{\partial \Phi(\hat{p}_i^k)}{\partial \hat{p}_i^k} = \frac{\partial}{\partial \hat{p}_i^k} \left( \log_2 \left( 1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k \right) \right)
\] 
\[+ \frac{\partial}{\partial \hat{p}_i^k} \left( \sum_{i \in N, j \neq i} \log_2 \left( 1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k \right) \right)
\]

Combining equations (3) and (28), we obtain
\[
    \frac{\partial \Phi(\hat{p}_i^k)}{\partial \hat{p}_i^k} = \frac{\hat{\alpha}_i^k (\hat{p}_i^k)^*}{1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k} + \sum_{i \in N, \forall j \neq i} \left( \frac{\partial \hat{\alpha}_i^k (\hat{p}_i^k)^*}{\partial \hat{p}_i^k} \hat{p}_i^k \right)
\] 
(28)

VIII. APPENDIX B

In this section, the solution of the optimization problem (14) is provided. The original problem is given by
\[
    \min \sum_{k \in L_i} \Delta p_i^k, i \in N_C
\]
\[s.t. \sum_{k \in L_i} \Delta c_i^k = \Delta r_i^\text{tar}, i \in N_C
\]

The Lagrangian dual function can be formulated as
\[
    L(\Delta p_i^k) = \sum_{k \in L_i} \Delta p_i^k + \lambda \left( \Delta r_i^\text{tar} - \sum_{k \in L_i} \Delta c_i^k (\Delta p_i^k) \right)
\] 
(29)

The derivative of (29) with respect to \( \Delta p_i^k \) is
\[
    \frac{\partial L(\Delta p_i^k, \Delta c_i^k)}{\partial \Delta p_i^k} = 1 - \lambda \frac{\partial \Delta c_i^k (\Delta p_i^k)}{\partial \Delta p_i^k}
\] 
(30)

where \( \Delta c_i^k (\Delta p_i^k) \) is given by (13). Hence, the derivative of \( \Delta c_i^k \) with respect to \( \Delta p_i^k \) is
\[
    \frac{\partial \Delta c_i^k}{\partial \Delta p_i^k} = \log_2 \frac{\hat{\alpha}_i^k (\hat{p}_i^k)^*}{1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot (\hat{p}_i^k + \Delta p_i^k)}
\] 
(31)

Based on (30) and (31), \( \lambda \) can be expressed as
\[
    \lambda = \ln 2 \cdot \frac{1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot (\hat{p}_i^k + \Delta p_i^k)}{\hat{\alpha}_i^k (\hat{p}_i^k)^*}
\] 
(32)

Therefore, the increase power \( \Delta p_i^k \) is given by
\[
    \Delta p_i^k = \left[ \log_2 \lambda - \frac{1}{\hat{\alpha}_i^k (\hat{p}_i^k)^*} \right]^{+}
\] 
(33)

Differentiating (29) with respect to \( \lambda \), we obtain
\[
    \frac{\partial L(\Delta p_i^k, \Delta c_i^k)}{\partial \lambda} = \Delta r_i^\text{tar} - \sum_{k \in L_i} \Delta c_i^k (\Delta p_i^k)
\] 
(34)

Taking (33) and (13) into (34) will result in
\[
    \Delta r_i^\text{tar} = \sum_{k \in L_i} \log_2 \left( \frac{\hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \log_2 \lambda}{1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k} \right)
\] 
(35)

If we define \( \beta_i^k := \frac{\hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \log_2 \lambda}{1 + \hat{\alpha}_i^k (\hat{p}_i^k)^* \cdot \hat{p}_i^k} \), (35) can be reduced to
\[
    \Delta r_i^\text{tar} = \sum_{k \in L_i} \log_2 \left( \beta_i^k \cdot \lambda \right).
\]
For transmitter \( i \), we define \( l(\hat{p}_i^k) = M_i \), which will lead to
\[
    M_i \cdot \log_2 \lambda + \sum_{k \in L_i} \log_2 \beta_i^k = \Delta r_i^\text{tar}
\] 
(36)

Apparently, \( \lambda \) can be expressed as
\[
    \lambda = \exp \left( \frac{\ln 2}{M_i} \left( \Delta r_i^\text{tar} - \sum_{k \in L_i} \log_2 \beta_i^k \right) \right)
\] 
(37)

The optimal data rate increase \( \Delta c_i^k \) can be derived as
\[
    \Delta c_i^k = \log_2 \left( \beta_i^k \cdot \exp \left( \frac{\ln 2}{M_i} \left( \Delta r_i^\text{tar} - \sum_{k \in L_i} \log_2 \beta_i^k \right) \right) \right)
\]
IX. APPENDIX C

The derivation of (17) is provided in this section. The original problem is given by

$$\min \sum_{k \in L_i} \Delta c^k_i, \forall i \in \mathcal{N}_D$$

subject to

$$\sum_{k \in L_i} \Delta c^k_i = \Delta p^* \cdot r, \forall i \in \mathcal{N}_D \tag{38}$$

Since $\hat{\alpha}_i^k (\hat{p}^*_{i})$ and $\hat{p}^*_{i}$ are constants, based on (16), problem (38) can be reduced to

$$\max \sum_{k \in L_i} \log_2 \left( 1 + \frac{\hat{\alpha}_i^k (\hat{p}^*_{i})}{\Delta p^*} \right) \left( \hat{p}^*_{i} - \Delta p^k_i \right)$$

subject to

$$\sum_{k \in L_i} \Delta p^k_i = \Delta p^* \cdot r, \forall i \in \mathcal{N}_D \tag{39}$$

The Lagrangian dual function can be formulated as

$$L(\Delta p^k_i) = \sum_{k \in L_i} \log_2 \left( 1 + \frac{\hat{\alpha}_i^k (\hat{p}^*_{i})}{\Delta p^*} \right) \left( \hat{p}^*_{i} - \Delta p^k_i \right) + \lambda \left( \Delta p^* \cdot r - \sum_{k \in L_i} \Delta p^k_i \right) \tag{40}$$

The derivative of (40) with respect to $\Delta p^k_i$ is

$$\frac{\partial L(\Delta p^k_i)}{\partial \Delta p^k_i} = \log_2 \left( 1 + \frac{\hat{\alpha}_i^k (\hat{p}^*_{i})}{\Delta p^*} \right) \frac{\hat{\alpha}_i^k (\hat{p}^*_{i})}{\Delta p^*} - \lambda$$

(41)

Thus, $\Delta p^k_i$ can be expressed with parameter $\lambda$ as

$$\Delta p^k_i = \left[ \frac{1}{\hat{\alpha}_i^k (\hat{p}^*_{i})} + \frac{1}{\Delta p^*} \right]^{+} \left( \frac{1}{\hat{\alpha}_i^k (\hat{p}^*_{i})} - \frac{1}{\ln 2 \cdot \lambda} \right) \tag{42}$$

From the constraint $\sum_{k \in L_i} \Delta p^k_i = \Delta p^* \cdot r$, $\lambda$ is

$$\lambda = \frac{M_i}{\sum_{k \in L_i} \Delta p^k_i + \sum_{k \in L_i} \frac{1}{\hat{\alpha}_i^k (\hat{p}^*_{i})} - \Delta p^* \cdot r} \ln 2 \tag{43}$$

where $(\hat{p}^*_{i}) = M_i$. Taking (43) into (42), because $\Delta p^* \cdot r = p^*_{i} - \sum_{k \in L_i} \Delta p^k_i$, we obtain

$$\Delta p^k_i = \left[ \frac{1}{\hat{\alpha}_i^k (\hat{p}^*_{i})} - \frac{\lambda}{M_i} \right]^{+} \tag{44}$$

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